Alishev Abdumannon (Jizzax davlat Pedagogika universiteti dotsenti, f.-m.f.n.),
Baxtiyorova Kamola (Jizzax davlat Pedagogika universiteti 3-kurs talabasi)

BA'ZI INTEGRO-DIFFERENSIAL TENGLAMALAR SISTEMASINING ASIMPTOTIK INTEGRALLASH

Annotatsiya. *Ushbu maqolada sekin o'zgaruvchi koeffitsiyentli integro-differensial tenglamalar sistemasi uchun asimptotik yechimlar tuzilgan. Ya'ni, ishda sekin o'zgaruvchi koeffitsiyentli chiziqli bo'lmagan integro-differensial tenglamalar sistemasining xarakteristik tenglamasining bitta ildizi aynan nolga teng, qolganlari noldan farqli bo'lgan kritik hol uchun asimptotik yechimini tuzish qaralgan.*

Kalit so'zlar: *xarakteristik tenglama, umumlashtirilgan teskari matritsa, vektor matritsa, skalyar ko'paytma.*

АСИМПТОТИЧЕСКОЕ ИНТЕГРИРОВАНИЕ СИСТЕМЫ НЕКОТОРЫХ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

Аннотация. *В данной статье построены асимптотические решения системы интегро-дифференциальных уравнений с медленно изменяющимися коэффициентами. Рассматривается построение асимптотического решения для критического случая, когда один из корней характеристического уравнения нелинейной системы интегро-дифференциальных уравнений равен нулю, а остальные отличны от него.*

Ключевые слова: *характеристические уравнения, обобщённо обратная матрица, векторная матрица, скалярное произведение.*

ASYMPTOTIC INTEGRATION OF A SYSTEM OF CERTAIN INTEGRO-DIFFERENTIAL EQUATIONS

Annotation. *This article constructs asymptotic solutions for a system of integro-differential equations with slowly varying coefficients. The study considers the construction of an asymptotic solution for the critical case where one root of the characteristic equation of the nonlinear system of integro-differential equations is exactly zero, while the remaining roots are nonzero.*

Key words: *characteristic equation, generalized inverse matrix, vector matrix, scalar product.*

Kirish. Sekin o'zgaruvchi koeffitsiyentli differensial tenglamalar sistemasiga ko'plab differensial tenglamalar keltiriladi. Masalan, bir jinsli chegaraviy

$$y(a) = y(b) = 0 \tag{1}$$

shartni qanoatlantiruvchi

$$\frac{d^2y}{dx^2} + \big(\lambda g(x) - \tau(x)\big) = 0\,, \tag{2}$$

bunda λ −katta parameter, $g(x), r(x), [a, b]$ kesmada berilgan funksiyalar, Shturm-Liuvilli tenglamasida

$$x = \tau = \varepsilon t\ \varepsilon = \frac{1}{\sqrt{\lambda}} \tag{3}$$

almashtirishni bajarsak, undan

$$\frac{d^{\overline{2}y}}{dt^2} + (g(\tau) + \varepsilon^2\, r(\tau)y = 0\,, \tag{4}$$

ko'rinishidagi sekin o'zgaruvchi $g(\tau), r(\tau)$ koeffitsiyentli differensial tenglamaga ega bo'lamiz.

Sekin o'zgaruvchi koeffitsientli chiziqli differensial tenglamalar sistemasini o'rganish o'tgan asrning 50-yillarida S.F.Feshenkoning ishlari paydo bo'lgandan boshlagan. S.F.Feshenko tomonidan

$$\frac{d^2y}{dt^2} + \varepsilon p(\tau, \varepsilon)\frac{dy}{dt} + g(\tau, \varepsilon)y = \varepsilon f(\tau, \varepsilon)e^{i\theta(\tau,\varepsilon)}, \tag{5}$$

ko'rinishidagi ikkinchi tartibli differensial tenglamaning asimptotik yechimini tuzish usulini taklif etgan, bunda $p(\tau, \varepsilon), g(\tau, \varepsilon)$ sekin o'zgaruvchi funksiyalar, $\tau = \varepsilon t$, $\varepsilon > 0$ kichik parametr.

Shunga o'xshash S.F.Feshenko tomonidan

$$A(\tau, \varepsilon)\frac{d^2x}{dt^2} + \varepsilon C(\tau, \varepsilon)\frac{dx}{dt} + B(\tau, \varepsilon)x = f(\tau, \varepsilon)e^{i\theta(t,\varepsilon)} \tag{6}$$

differensial tenglamalar sistemasi ustida tadqiqot ishlarini olib borgan.

Sekin o'zgaruvchi koeffitsiyentli differensial tenglamalar sistemasining xarakteristik tenglamasining ildizlari karrali bo'lgan hollar N.I.Shkil [1,2] ishlarida o'rganilgan.

Yuqori tartibli hosila oldida parametr qatnashgan differensial tenglamalarni ham sekin o‘zgaruvchi koeffitsiyentli differensialli tenglamalar sistemasini keltirish mumkin.

Masalan,

$$\varepsilon\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+g(x)y=0\,, \tag{7}$$

tenglamada

$$y=\varepsilon t,\frac{dy}{dx}=z_2,x=\tau=\varepsilon t\,, \tag{8}$$

almashtirish olib, birinchi tartibli

$$\frac{dz}{dt}=A(\tau,\varepsilon)z \tag{9}$$

differensial tenglamalar sistemasini hosil qilamiz, bunda

$$A(\tau,\varepsilon)=A_0(\tau)+\varepsilon A_1(\tau,\varepsilon),\ A_0(\tau)=\begin{pmatrix}0 & 1\\ 0 & -p(\tau)\end{pmatrix},$$

$$A_1(\tau)=\begin{pmatrix}0 & 0\\ -q(\tau) & 0\end{pmatrix},\ z=\begin{pmatrix}z_1\\ z_2\end{pmatrix}.$$

Bu olimlarning o‘quvchilari, asosan, chiziqli differensal tenglamalar sistemasi ustida ilmiy tadqiqot ishlarini olib borgan. Ushbu ishda ularning ishlaridan farqli chiziqli bo‘lmagan integro-differensial tenglamalar sistemasining xarakteristik tenglamasining bitta ildizi aynan nolga teng bo‘lgan kritik hol uchun asimptotik yechimini tuzishga umumlashgan teskari matritsa usuli qo‘llanilgan. Sekin o‘zgaruvchi koeffitsiyentli differensial va differensial tenglamalar sistemasiga astrofizika, avtomatik tartibga keltirish, optimal boshqarish, gidrotexnikaning ko‘pgina masalalari keltirilishini e’tiborga olsak, bu yo‘nalish ustida ilmiy tadqiqot ishlarini olib borish muhimdir.

Masalaning qo‘yilishi. Ushbu ishda

$$\varepsilon^{\frac{p}{q}}\frac{dx}{dt}=A(\tau,\varepsilon)x+\varepsilon^2\int_0^L\mathcal{K}(\tau,\sigma,\varepsilon)x(s,\varepsilon)ds+\varepsilon f(\tau,x,\varepsilon)\,, \tag{10}$$

ko‘rinishidagi sekin o‘zgaruvchi koeffitsiyentli chiziqli bo‘lmagan integro-differensial tenglamalar sistemasining yaqinlashuvchi yechimlarini tuzish masalasini qaraymiz, bunda $x(t,\varepsilon)$ –noma’lum, berilgan $n-$ o‘lchovli $A(\tau,\varepsilon),\mathcal{K}(\tau,\sigma,\varepsilon)$ kvadrat matritsalar $\varepsilon>0$ kichik parametrning darajalari bo‘yicha yaqinlashuvchi darajali qatorga yoyiladi:

$$A(\tau,\varepsilon)=\sum_{m=0}^{\infty}\varepsilon^m A_m(\tau)\,,\mathcal{K}(\tau,\sigma,\varepsilon)=\sum_{m=0}^{\infty}\varepsilon^m\mathcal{K}_m(\tau,\sigma), \tag{11}$$

$f(\tau,x,\varepsilon)$ –vektor $(\tau,x_0,0)$ nuqta atrofida Teylor qatoriga yoyiladi: $\tau=\varepsilon t\,,\sigma=\varepsilon s$

$\tau\in[0,L]$ oraliqda sekin o‘zgaruvchi bo‘ladi, p va q o‘zaro tub sonlar.

(10)-sistemasining asimptotik yechimi uning

$$\det[A_0(\tau)-\lambda(\tau)\mathrm{E}]=0, \tag{12}$$

(E – birlik matritsa) xarakteristik tenglamasining ildizlariga bog‘liq.

(12) – tenglamaning ildizlaridan bittasi aynan nolga teng bo‘lib, qolganlari noldan farqli bo‘lgan [3,4] kritik hol uchun formal xususiy yechimini tuzish va bu yechimning asimptotik xarakterga ega ekanligi isbotlash masalasini qaraymiz.

(10)-sistemaning yechimini ko‘rinishi p va q sonlar orasidagi munosabatlarga bog‘liq bo‘lgan $p>q$ va $p<q$ hollar uchun yechimlarni tuzishga umumlashgan teskari matritsa usulini qo‘llaymiz.

Yechimni tuzish algoritmi. $p<q$ bo‘lgan hol uchun (10)-sistemaning asimptotik yechimini quyidagi teorema ko‘rsatadi.

1-teorema. Faraz qilaylik, quyidagi shartlar bajarilsin:

a) $A_m(\tau)(\,m=0,1,2,\dots)$ matritsalar $\tau\in[0,L]$ o‘zgaruvchi bo‘yicha cheksiz differensiallanuvchi, (10)-tenglamaning yadrosi $\mathcal{K}_m(\tau,\sigma)(m=1,2,\dots)$ matritsalar $\forall\tau,\sigma\in[0,L]$lar uchun τ o‘zgaruvchi bo‘yicha cheksiz uzliksiz xususiy hosilalarga ega bo‘lsin;

b) $f(\tau,x,\varepsilon)$ vektor funksiya $p(\tau,x)\times(0<\varepsilon\le\varepsilon_0)$ sohada cheksiz differensiallanuvchi, bunda $p(\tau,x)-\ \tau$ va x o‘zgaruvchilar fazosining ba’zi sohasi;

d) $\tau\in[0,L]$ kesmada $A_0(\tau)$ –matritsaning xos qiymatlari $\lambda_1(\tau)\equiv 0$,

$\mathcal{R}e\lambda_j(\tau)\le 0\ (j=\overline{2,n})$ shartni qanoatlantirsin. U holda (10)-sistema

$$x(t,\varepsilon)=\sum_{m=0}^{\infty}\mu^m u_m(\tau),\mu=\sqrt[q]{\varepsilon} \tag{13}$$

ko‘rinishidagi formal xususiy yechimga ega.

Isbot. Teoremani isbotlash qilish uchun (13)-qatorni (10)-sistemaga qo‘yib, $f\left(\tau,x(\tau,\mu,\mu^q)\right)$ vektor-funksiyani $(\tau,u_0,0)$ nuqta atrofida Teylor qatoriga yoyilmasiga asosan,

$$\mu^p u'(\tau,\mu) = A(\tau,\mu^q)u(\tau,\mu) + \mu^{2q}\int_0^T \mathcal{K}(\tau,\sigma,\mu^q)u(s,\varepsilon)ds + \mu^q\{f(\tau,u_0,0) + \mu f_u(\tau)u_1 + \cdots + \mu^m[f_u(\tau)u_m(\tau) + f_{m-1}(\tau) + \tilde{f}_{m-1-q}(\tau)] + \cdots\} \quad (14)$$

ayniyatni hosil qilamiz, bunda $f_u(\tau) = \frac{\partial f^i}{\partial u^a}$ matritsaning elementlari va $f_1(\tau) = \frac{\partial f}{\partial \mu}$ vektorning koordinatalari $(\tau, u_0(\tau), 0)$ nuqtada hisoblanadi, $f_u(\tau)$ –vektor $u_m(\tau)$ $(m = 1,2,\dots)$ vektorlar orqali ifodalanadi.

(14) – ayniyatdan μ parametrning bir xil darajalari oldidagi koeffitsiyentlarini tenglashtirib (10)- qatorning noma'lum hadlarini aniqlash uchun

$$A_0(\tau)u_j(\tau)=0\ j = 0,1,\dots,q-1, \quad (15)$$

$$A_0(\tau)u_q(\tau)=-A_1(\tau)u_0(\tau) - \mathrm{f}(\tau,u_0(\tau),0),\dots \quad (16)$$

$$A_0(\tau)u_m(\tau)= -A_1(\tau)u_{m-1}(\tau) - f_u(\tau)u_{m-1}(\tau)\sum_{i=2}^{\left[\frac{m}{q}\right]} A_i(\tau)\,u_{m-qi}(\tau) - f_{m-1}(\tau) + F_m(\tau)\,, \quad (17)$$

bunda

$$F_m(\tau) = u'_{m-p}(\tau) - \sum_{i=2}^{\left[\frac{m}{q}\right]} A_i(\tau)\,u_{m-qi}(\tau) - \int_0^T \sum_{i=0}^{\left[\frac{m}{2q}\right]} \mathcal{K}_m(\tau,\sigma)u_{m-(2q-1)i}(s)ds - \tilde{f}_{m-q+2p-1}(\tau)\,, m = q, q+1,\dots$$

($[\frac{m}{q}]$, $\frac{m}{q}$ sonning butun qismi) tenglamalar sistemasiga ega bo'lamiz.

(13)-qatorning noma'lum hadlarini aniqlash uchun (15) - (17) tenglamalarni yechimga ega ekanligini ko'rsatamiz.

(15)-tenglamadan [5] ga asosan,

$$u_j(\tau) = \varphi\eta_j(\tau), j = 0,1,\dots,q-1, \quad (18)$$

yechimlarni aniqlaymiz, bunda $\eta_j(\tau)$, $(j = 0,1,\dots,q-1)$ keyingi qadamlarda aniqlanadigan noma'lum funksiyadir.

(16)-tenglamani (18) tenglamani nazarda tutib,

$$A_0(\tau)u_q(\tau)= -A_1(\tau)\varphi\eta_0(\tau) - f(\tau,\psi\eta_0(\tau),0) \quad (19)$$

ko'rinishida yozamiz. Bu tenglama yechimga ega bo'lish uchun uning o'ng tomoni $A_0(\tau)$ matritsaning qo'shmasi bo'lgan $A_0^*(\tau)$ matritsaning xos qiymatining transponerlangan ψ vektorga ortogonal bo'lishi zarur va yetarlidir:

$$\big(\psi, A_1\varphi\eta_0(\tau)\big) + \Big(\psi, f\big(\tau,\varphi\eta_0(\tau)\big)\Big) = 0$$

yoki

$$(\psi, A_1(\varphi)\eta_0(\tau) + \Big(\psi, f\big(\tau,\varphi\eta_0(\tau)\big)\Big) = 0 \quad (20)$$

(20)-tenglama $\eta_0(\tau)$-noma'lum funksiyaga nisbatan oshkormas, qulaylik uchun uni

$$Q(\tau,\varphi\eta_0(0) = 0 \quad (21)$$

ko'rinishida yozamiz. Natijada $\eta_0(\tau)$ noma'lum funksiyaga nisbatan oshkormas tenglamani hosil qildik. Ushbu tenglama [6] – ishga asosan, oshkormas tenglama haqidagi teoremaning barcha shartlarini qanoatlantiradi, bu har bir aniq hol uchun tekshiriladi va undan $\eta_0(\tau)$ funksiya aniqlanadi.

(19)- tenglama uchun yechim mavjud bo'lish sharti bajarildi, u holda undan

$$u_q(\tau) = -A_0^+(\tau)[\,A_1\big(\tau\big)\varphi\eta_0(\tau) + f\big(\tau,\varphi\eta_0(\tau)\big)\big] + \varphi\eta_q(\tau) \quad (22)$$

vektorni aniqlaymiz, bunda $\eta_q(\tau)$ –keyingi qadamda aniqlanadigan noma'lum funksiya, $A_0^+(\tau)$, $A_0(\tau)$ matritsaga nisbatan umumlashgan teskari matritsa,

$$A_0^+(\tau) = [A_0(\tau) - (\psi\otimes\varphi)]^{-1} + (\varphi\otimes\psi) \quad (23)$$

ko'rinishga ega, bunda $\otimes$ vektorlarni tenzor ko'paytmasi belgisi.

Shu jarayonni davom ettirib, m-chi qadamda (17) tenglamani noma'lum funksiyalar aniqlangan deb

$$A_0(\tau)u_m(\tau)=-[A_1(\tau) - f_u(\tau)]\varphi\eta_{m-1}(\tau) - f_{m-1}(\tau) - F_m(\tau) \quad (24)$$

ko'rinishda yozamiz. Bu tenglama uchun yechim mavjud bo'lish sharti

$$(\psi,(A_1(\tau)-f_u(\tau)\varphi)\eta_{m-1}(\tau)+(\psi,\left(f_{m-1}(\tau)-F_m(\tau)\right)=0 \quad (25)$$

ko‘rinishida bo‘ladi. (28)-tenglamadan $\eta_{m-1}(\tau)$ noma’lum funksiya

$$\eta_{m-1}(\tau)=\frac{(\psi,(F_m(\tau)-f_{m-1}(\tau)))}{(\psi,(A_1(\tau)+f_u(\tau))\varphi)} \quad (26)$$

bo‘lishini aniqlaymiz. (25) shart (24) – tenglama uchun o‘rinli ekanligini nazarda tutib undan (13)-qatorning m-chi noma’lum vektorni aniqlaymiz:

$$u_m(\tau)=-A_0^{+}(\tau)[A_1(\tau)+f_u(\tau))\varphi\eta_{m-1}-f_{m-1}(\tau)+F_m(\tau)]+\varphi\eta_m(\tau) \quad (27)$$

bunda $\eta_m(\tau)$-keyingi qadamda aniqlanadigan noma’lum funksiya, shunday qilib (15)-(17) tenglamalar sistemasidan (13) qatorning $u_m(\tau)$ $(m=0,1,2,\dots)$ vektorlarni aniqladik. Bu vektorlar teoremaning shartiga asosan [0, L], kesmada istalgan tartibli hosilalarga ega. Teorema isbotlandi.

Formal xususiy yechimning asimptotik xossasi. Tuzilgan (13)-yechimni asimptotik xarekterga ega ekanligini ko‘rsataylik, buning ma’nosi, agar formal qatorning dastlabki k −ta hadini olib hosil qilingan qismiy yig‘indi x_k (k −ta yaqinlashuvchi) vektorni tuzsak, bu vektor tayinlangan k va $\varepsilon\longrightarrow 0$ ga (10)- sistemaning aniq yechimiga intiladi. k −yaqinlashuvchi yechim

$$x_k(t,\varepsilon)=\sum_{m=0}^{k}\varepsilon^s u_m(\tau) \quad (28)$$

vektordan iborat.

Lemma. Agar birinchi teoremaning barcha shartlari bajarilsa, u holda k −yaqinlashuvchi $x_k(t,\varepsilon)$ yechim

$$\frac{dx_k}{dt}=A(\tau,\varepsilon)x_k+\varepsilon^s\int_0^T\mathcal{K}(\tau,\sigma,\varepsilon)x_k(s,\varepsilon)ds+\varepsilon f(\tau,x_m,\varepsilon)+\varepsilon^{\kappa+1}R_k(\tau,\varepsilon) \quad (29)$$

integro-differensial tenglamani qanoatlantiradi, bunda $R_k(\tau,\varepsilon)$- vektor funksiya $[0,L]$ kesmada chegaralangan qoldiq had.

Lemmani isbot qilish uchun (28)-ni (10)-sistemaga qo‘yamiz, birinchi teoremaga asosan, (28)-vektor (10) -sistemani $0(\varepsilon^{\kappa+1})$ aniqlikda qanoatlantirishiga ishonch hosil qilamiz.

Faraz qilaylik, boshlang‘ich

$$V(\tau,\varepsilon)|_{\tau=0}=V_0\,, \quad (30)$$

shartni qanoatlantirsin.

$$\frac{dV}{dt}=A_0(\tau)V \quad (31)$$

tenglama $V(\tau,\varepsilon)$ yechimga ega bo‘lib, $\forall\tau\in[0,L]$ uchun

$$\|V(\tau,\varepsilon)\|\le M \quad (32)$$

tengsizlikni qanoatlantirsin, bunda M, ε −ga bog‘liq bo‘lmagan o‘zgarmas.

1-teoremada tuzilgan yechimni asimptotik xarakterga ega ekanligini quyidagi teorema ko‘rsatadi.

2-teorema. Faraz qilaylik, (10) sistema uchun 1-teoremaning shartlari bajarilsin va:

a) $f(\tau,x,\varepsilon)$ −vektor funksiya o‘zgarmas l soni uchun Lipshits shartini qanoatlantirsin:

$$\|f(\tau,x,\varepsilon)-f(\tau,x_k,\varepsilon)\|\le l\|x-x_k\|\,; \quad (33)$$

b) (32)-shart bajarilib,

$$x(t,\varepsilon)|_{t=0}=x_k(t,\varepsilon)|_{t=0} \quad (34)$$

tenglik o‘rinli bo‘lsin. U holda har qanday katta $L>0$ va o‘zgarmas $\varepsilon>0$ ga bog‘liq bo‘lmagan o‘zgarmas C uchun shunday $\varepsilon\in(0,\varepsilon_0]$ −ni mos qo‘yish mumkinki,

$0\le t\le\frac{L}{\varepsilon}$ oraliqda

$$\|x(t,\varepsilon)-x_k(t,\varepsilon)\|\le\varepsilon^{k+1-q}C\,, \quad (35)$$

tengsizlik o‘rinli boladi.

Isbot. (10)-sistemadan (29)-sistemani oshirib,

$$\frac{dz}{dt}=A_0(\tau)z+\varepsilon A_1(\tau,\varepsilon)z+\varepsilon^s\int_0^T P_k(\tau,\sigma)z(s,\varepsilon)ds+\varepsilon[f(\tau,x,\varepsilon)-f(\tau,x_k,\varepsilon)]+\varepsilon^k R_k(\tau,\varepsilon) \quad (36)$$

tenglamani hosil qilamiz, bunda

$$z(t,\varepsilon)=x(t,\varepsilon)-x_k(t,\varepsilon) \quad (37)$$

$$A_1(\tau,\varepsilon)=\sum_{k=1}^{\infty}\varepsilon^k A_k(\tau)\,,P_k(\tau,\sigma)=K(\tau,\sigma)-K_k(\tau,\varepsilon)$$

(36)-tenglama

$$z(t,\varepsilon)=\varepsilon\int_0^t\{V(\tau,\varepsilon)\cdot V^{-1}(\tau,\varepsilon)z(t,\varepsilon)dt+\varepsilon^2\int_0^t(\int_0^T P_k(\tau,\sigma)V(\tau,\varepsilon)V^{-1}(\tau,\varepsilon)z(s,\varepsilon)ds)dt+$$
$$\varepsilon[f(\tau,x,\varepsilon)-f(\tau,x_k,\varepsilon)]\}dt_1+\varepsilon^{k+1}\int_0^t V(\tau,\varepsilon)\cdot V^{-1}(\tau,\varepsilon)\,R_k(\tau,\varepsilon)dt_1 \quad (38)$$

integral tenglamaga ekvivalent.

(38)-tenglamadan norma olib, quyidagi tengsizlikka ega bo‘lamiz:

$$\|z(t,\varepsilon)\|=\varepsilon\left\|\int_0^T V(\tau,\varepsilon)\right\|_T\|V^{-1}(\tau,\varepsilon)\|\|A_1(\tau,\varepsilon)\|\|z(t,\varepsilon)\|dt$$
$$+\varepsilon^2\int_0^t(\int_0^T\|P_k(\tau,\sigma)\|\,ds)\|z(t,\varepsilon)\|dt\le\varepsilon(AN+lM)\int_0^t\|z(t,\varepsilon)\|$$
$$dt+\varepsilon^2\int_0^T H\|z(L,\varepsilon)\|\,dt+\varepsilon^{k+1-q}C, \quad (39)$$

tengsizlikni hosil qilamiz (39) – ifodaga Gronuolla – Bellman [7] tengsizligini qo‘llab, quyidagi tengsizlikni hosil qilamiz :

$$\|z(t,\varepsilon)\|=\|x(t,\varepsilon)-x_k(t,\varepsilon)\|\le\varepsilon^{k+1-q}C, \quad (40)$$

Bunda:

$C=MLRexp(HN-lM),[0,L]$ kesmada

$\|A_1(\tau,\varepsilon)\|\le N,\|R_k(\tau,\varepsilon)\|\le R,\|P_k(\tau,\sigma)\|\le H$ tengsizliklar o‘rinli.

2-teorema isbotlandi.

Xulosa. Ushbu ishda parametrga bog‘liq sekin o‘zgaruvchi koeffitsiyentli integro-differensial tenglamalar sistemasining, hosila oldidagi parametrning darajasiga bog‘liq, $p<q$ va $p>q$ hollarining $p<q$ xoli uchun formal xususiy yechim tuzilib, uning asimptotik xossaga ekanligi isbotlandi. Ishning hajmi kattarib ketishini e’tiborga olib, $p>q$ bo‘lgan hol uchun yechimni tuzish algoritmi $p<q$ holga o‘xshashligini nazarda tutib shu hol maqolaga kiritilmadi. $p<q$ bo‘lgan holda noma’lum $\eta_i(\tau)$ $(i=0,1,2,\dots,)$ funksiyani aniqlash uchun algebraik tenglamalar hosil bo‘ladi. $p>q$ bo‘lgan hol uchun esa birinchi tartibli chiziqli integro-differensial tenglamalar hosil bo‘ladi.

1. Eslatma. Ishda qaralayotgan sistemaning xarakteristik tenglamasi k –karrali ildizga ega bo‘lgan, ya’ni $A_0(\tau)$ matritsa k –karrali xos qiymatga ega bo‘lib, unga bitta elementar bo‘luvchi mos kelgan hol.

2. Eslatma. Qaralayotgan sistemaning xarakteristik tenglamasi bitta n karrali ildizga ega bo‘lib, unga shunday o‘lchamli bitta elementar bo‘luvchi mos kelgan hollar uchun asimptotik yechimlar tuzish masalasi o‘rganiladi. Chunki ikkala hol uchun ham fan va texnikaning ko‘plab masalalari keltiriladi. Shuning uchun ham bunday hollar ustida ilmiy tadqiqot ishlarini olib borish muhimdir.

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