важное значение при моделировании процессов аномальной диффузии и, в отличие от классических уравнений диффузии, обладают более сложными физическими и математическими свойствами.

Ключевые слова: *дробная производная, производная Римана–Лиувилля, уравнение супердиффузии, начально–краевая задача, принцип Дюамеля, функция Грина, функция Миттага–Леффлера, аномальная диффузия.*

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SOLUTION OF AN INITIAL–BOUNDARY VALUE PROBLEM FOR A ONE-DIMENSIONAL SUPERDIFFUSION EQUATION

Annotation. *In this paper, the problem of solving an initial–boundary value problem for a one-dimensional superdiffusion equation is studied in detail. The study considers a superdiffusion equation involving a fractional derivative of the Riemann–Liouville type with respect to the time variable. Such equations are of great importance in modeling anomalous diffusion processes and, unlike classical diffusion equations, possess more complex physical and mathematical properties.*

Keywords: *fractional derivative, Riemann–Liouville derivative, superdiffusion equation, initial–boundary value problem, Duhamel's principle, Green's function, Mittag–Leffler function, anomalous diffusion.*

Kirish. So'nggi yillarda kasr tartibli differensial tenglamalar nazariyasi matematik fizika, muhandislik va tabiiy fanlarning ko'plab sohalarida muhim tadqiqot yo'nalishlaridan biriga aylandi. Xususan, anomal diffuziya jarayonlarini tavsiflashda klassik diffuziya tenglamalari yetarli bo'lmay, vaqt yoki fazo bo'yicha xotira effektlarini hisobga oluvchi kasr tartibli modellar keng qo'llanilmoqda. Bunday jarayonlar plazma fizikasi, biologik tizimlar, moliyaviy modellashtirish hamda murakkab muhitlarda zarrachalar transporttini o'rganishda uchraydi.

Superdiffuziya tenglamalari anomal diffuziyaning muhim sinfi bo'lib, ular zarrachalarning o'rtacha kvadratik siljishi vaqtga nisbatan tezroq o'sishi bilan tavsiflanadi. Ushbu tenglamalarda, ko'pincha, Riman –Liuvill yoki Kaputo tipidagi kasr tartibli hosilalar ishtirok etadi. Natijada yechimlarning mavjudligi, yagonaligi va aniqligi masalalari klassik holatga nisbatan ancha murakkablashadi.

Mazkur maqolada vaqt o'zgaruvchisi bo'yicha Riman–Liuvill tipidagi kasr tartibli hosilani o'z ichiga olgan bir o'lchamli superdiffuziya tenglamasi uchun qo'yilgan boshlang'ich–chegaraviy masala tadqiq etiladi. Masalani yechishda Dyuamel prinsipi, Furye va Laplas almashtirishlari hamda Grin funksiyasi usullaridan foydalaniladi. Olingan natijalar superdiffuziya tenglamalari uchun klassik yechimni aniq ko'rinishda ifodalash imkonini beradi.

Mavzuga oid adabiyotlar tahlili. Kasr tartibli differensial tenglamalar nazariyasining asoslari I. Podlubny tomonidan yozilgan fundamental monografiyada batafsil bayon etilgan bo'lib, unda Riman–Liuvill va Kaputo hosilalari, ularning xossalari hamda amaliy qo'llanilishlari keng yoritilgan. Ushbu manba kasr tartibli modellar bilan ishlashda nazariy asos sifatida xizmat qiladi.

R.Gorenflo va hammualliflarining ishlarida Mittag–Leffler funksiyalari, ularning analitik xossalari va kasr tartibli tenglamalar yechimlaridagi roli chuqur o'rganilgan. Aynan ikki parametrli Mittag–Leffler funksiyasi superdiffuziya tenglamalarining fundamental yechimlarini ifodalashda muhim ahamiyatga ega.

R.Hilfer tomonidan olib borilgan tadqiqotlarda kasr tartibli hosilalarga kirish, ularning fizik ma'nosi va turli modellashtirish masalalaridagi o'rni tahlil qilingan. Ushbu ishlar anomal diffuziya tushunchasining shakllanishida muhim o'rin tutadi. Shuningdek, Shinaliyev, Turmetov va Umarovlarning maqolalarida kasr tartibli differensial tenglamalar uchun yechimlarni qurishning samarali algoritmik usullari taklif etilgan. Ushbu yondashuvlar superdiffuziya va unga yaqin sinfdagi tenglamalar uchun aniq va sifatli yechimlar olish imkonini beradi.

Yuqoridagi adabiyotlar tahlili shuni ko'rsatadiki, kasr tartibli superdiffuziya tenglamalari nazariy va amaliy jihatdan dolzarb bo'lib, mazkur maqolada olingan natijalar ushbu yo'nalishdagi mavjud tadqiqotlarni mantiqiy ravishda to'ldiradi.

Ushbu maqolada vaqt o'zgaruvchisi bo'yicha kasr tartibli

$$D_t^{\alpha} u - a^2 u_{xx} = f(x,t), x \in R, t > 0 \quad (1)$$

superdiffuziya tenglamasinini Dyuamel prinsipi yordamida

$$I_{0+}^{1-\alpha} u \Big|_{t=0} = \varphi(x), x \in R \quad (2)$$

Boshlang'ich shartni qanoatlantiruvchi $u = u(x,t)$ funksiyalarini topish masalasini qaraymiz. Berilgan (1) $-$(2) masalada $0 < \alpha < 1$, D_t^{α} $-$Riman$-$Liuvill differensial operatori

$$D_t^{\alpha} v(t) = \frac{\partial}{\partial t} I_{0+}^{1-\alpha} v = \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial t} \int_0^t (t-s)^{-\alpha} v(s)\, ds$$

va

$$I_{0+}^{1-\alpha} v(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{1-\alpha} v(s)\, ds$$

Shuningdek, (1) −(2) to‘g‘ri masalada $\{f, \varphi\}$ −berilgan yetarlicha silliq funksiyalar deb qaraladi.

$u = \bar{u} + \overline{w}$ bo‘lsin.

Dastlab, (1) −(2) Koshi masalasini tatbiq etamiz, buning uchun (1) −(2) masalaga mos bir jinsli tenglamasi

$$\begin{cases} D_t^{\alpha} \bar{u} - a^2 \bar{u}_{xx} = 0, 0 < \alpha < 1, x \in R, t > 0 & (3) \\ I_{0+}^{1-\alpha} \bar{u} \,\big|_{t=0} = \varphi(x)\ x \in R & (4) \end{cases}$$

Koshi sharti bilan birgalikda qaraymiz.

1-teorema. Faraz qilaylik, $\varphi \in C_b(R)$ bo‘sin, u holda (3) −(4) masala yagona klassik yechimga ega bo‘lib, bu yechim

$$\bar{u}(x, t) = \int_R G(x-y, t)\varphi(y)dy, t > 0, x \in R \quad (5)$$

ko‘rinishda bo‘ladi, bu yerda:

$$G(x,t) = \frac{t^{\alpha-1}}{2\pi} \int_R e^{-i\xi x} \mathrm{E}_{\alpha,\alpha}(-a^2\xi^2 t^{\alpha}) d\xi \ \ t > 0, x \in R$$

$\mathrm{E}_{\alpha,\alpha}$ − ikki parametrli Mittag-Leffler funksiyasi. 1-eoremani isbotlashdan oldin $G(x,t)$ − Grin funksiyasining ba’zi xossalarini keltirib o‘tamiz.

1-lemma. $G(x,t)$ − Grin funksiyasi uchun quyidagilar o‘rinli:

$$D_t^{\alpha} G(x,t) - a^2 G_{xx}(x,t) = 0, t > 0, x \in R \quad (6)$$

$$\lim_{t \to +0} I_{0+}^{1-\alpha} G(x,t) = \varphi(x), x \in R \quad (7)$$

$$\int_R G(x,t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}, t > 0 \quad (8)$$

$$|D_t^{\alpha} G(x,t)| = a^2 |G(x,t)| \le C, t \ge \varepsilon > 0, \forall x \in R \quad (9)$$

bu yerda $\delta(x)$ − Dikarning deltasi:

$$\delta(x) = \begin{cases} +\infty, x = 0 \\ 0, x \ne 0 \end{cases} \bigwedge \int_R \delta(x - x_0)\varphi(x)dx = \varphi(x_0)$$

$\varphi(x) \in D(R)$ −asosiy funksiya.

Isbot. 1) dastlab, (7) tenglikning o‘rinli ekanligini ko‘rsatamiz. Buning uchun oldin $G(x,t)$ − Grin funksiyasining Riman-Liuvill kasr integralini olamiz.

$$I_{0+}^{1-\alpha} G(x,t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} Gx, s) ds =$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{s^{\alpha-1}}{2\pi} \int_R e^{-i\xi x} \mathrm{E}_{\alpha,\alpha}(-a^2\xi^2 s^{\alpha}) d\xi ds =$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{(t-s)^{-\alpha} s^{\alpha-1}}{2\pi} \int_R e^{-i\xi x} \sum_{m=0}^{\infty} \frac{(-a^2\xi^2)^m}{\Gamma(\alpha m + \alpha)} s^{\alpha m} d\xi ds =$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_R \frac{e^{-i\xi x}}{2\pi} \sum_{m=0}^{\infty} \frac{(-a^2\xi^2)^m}{\Gamma(\alpha m + \alpha)} d\xi \int_0^t (t-s)^{-\alpha} s^{\alpha-1} s^{\alpha m} ds$$

$$= \left| \begin{matrix} s = ty \\ s_1 = 0 \Rightarrow y_1 = 0 \\ s_2 = t \Rightarrow y_1 = 1 \\ ds = tdy \end{matrix} \right| =$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_R \frac{e^{-i\xi x}}{2\pi} \sum_{m=0}^{\infty} \frac{(-a^2\xi^2)^m}{\Gamma(\alpha m + \alpha)} d\xi \times \left[\int_0^1 t^{-\alpha} (1-y)^{-\alpha} t^{\alpha-1} y^{\alpha-1} t^{\alpha m+1} y^{\alpha n} dy \right] =$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_R \frac{e^{-i\xi x}}{2\pi} \sum_{m=0}^{\infty} \frac{(-a^2\xi^2)^m}{\Gamma(\alpha m+\alpha)} d\xi (t^{\alpha m} \int_0^1 (1-y)^{-\alpha} y^{\alpha(m+1)-1} dy) =$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_R \frac{e^{-i\xi x}}{2\pi} \sum_{m=0}^{\infty} \frac{(-a^2\xi^2)^m}{\Gamma(\alpha m+\alpha)} (t^{\alpha m} B(1-\alpha, \alpha(m+1)) d\xi =$$

$$= \frac{1}{\Gamma(1-\alpha)} \int_R \frac{e^{-i\xi x}}{2\pi} \sum_{m=0}^{\infty} \frac{(-a^2\xi^2)^m}{\Gamma(\alpha m+\alpha)} t^{\alpha m} \frac{\Gamma(1-\alpha)\Gamma(\alpha m+\alpha)}{\Gamma(\alpha m+1)} d\xi =$$

$$= \frac{1}{2\pi} \int_R e^{-i\xi x} \sum_{m=0}^{\infty} \frac{(-a^2\xi^2 t^\alpha)^m}{\Gamma(\alpha m+1)} d\xi = \frac{1}{2\pi} \int_R e^{-i\xi x} \mathrm{E}_\alpha(-a^2\xi^2 t^\alpha) d\xi.$$

Demak,

$$I_{0+}^{1-\alpha} G(x,t) = \frac{1}{2\pi} \int_R e^{-i\xi x} \mathrm{E}_\alpha(-a^2\xi^2 t^\alpha) d\xi. \tag{10}$$

Mittag-Leffler funksiyasining $E_\alpha(0) = 1$ xossasidan va Dikar funksiyasining fure almashtirishiga koʻra

$$\lim_{t \to +0} I_{0+}^{1-\alpha} G(x,t) = \delta(x), x \in R$$

boʻlishini koʻrish qiyin emas.

2) (8) tenglikni togʻridan togʻri integrallash yordamida koʻrsatamiz. Demak, $\forall\, t > 0$ uchun

$$\int_R G(x,t) = \int_R \frac{t^{\alpha-1}}{2\pi} \int_R e^{-i\xi x} E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) d\xi dx =$$

$$= \frac{t^{\alpha-1}}{2\pi} \int_R E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) \int_R e^{-i\xi x} d\xi dx =$$

$$= t^{\alpha-1} \int_R E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) (\frac{1}{2\pi} \int_R e^{-i\xi x} dx) d\xi =$$

$$= t^{\alpha-1} \int_R E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) \delta(\xi) d\xi = \frac{t^{\alpha-1}}{\Gamma(\alpha)}, t > 0, \tag{11}$$

Bu yerda biz $\lim_{\xi \to 0} E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) = \frac{1}{\Gamma(\alpha)}$ munosabatdan foydalandik;

3) endigi navbatda, hosilalarning (Riman-Liuvill kasr hosilasi va fazoviy oʻzgaruvchilar boʻyicha ikkinchi tartibli hosilalarning) mavjudligini koʻrsatamiz. Buning uchun, avvalo, formal ravishda hosila olamiz. $G(x,t)$ funksiyaning aniqlanishiga koʻra,

$$\frac{\partial^2}{\partial x^2} G(x,t) = \frac{t^{\alpha-1}}{2\pi} \int_R (-\xi^2) e^{i\xi x} E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) d\xi, \forall\, t > 0, x \epsilon R$$

Riman-Liuvill kasr hosilani hisoblashda ta'rifdan foydalanamiz:

$$D_t^\alpha G(x,t) = \frac{\partial}{\partial t} I_{0+}^{1-\alpha} G(x,t) = \frac{\partial}{\partial t} \left\{ \frac{1}{2\pi} \int_R e^{-i\xi x} E_\alpha(-a^2\xi^2 t^\alpha) d\xi \right\}$$

$$= \frac{1}{2\pi} \int_R e^{-i\xi x} \frac{\partial}{\partial t} E_\alpha(-a^2\xi^2 t^\alpha) d\xi = \left| \frac{d}{dt} E_\alpha(-\lambda t^\alpha) = -\lambda t^{\alpha-1} E_{\alpha,\alpha}(-\lambda t^\alpha), t > 0 \right|$$

$$= \frac{1}{2\pi} \int_R e^{-i\xi x} (-a^2\xi^2) t^{\alpha-1} E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) d\xi$$

$$= -\frac{a^2}{2\pi} t^{\alpha-1} \int_R e^{-i\xi x} \xi^2 E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) d\xi \tag{12}$$

(11) va (12) dan (7) tenglikning toʻgʻri ekanligini koʻrish qiyin emas;

4) agar ikki parametrli Mittag-Leffler funksiyasi uchun $0 < \alpha < 1, z > 0$ boʻlganda,

$$E_{\alpha,\alpha}(-z) \le \frac{c}{(1+z)^2}$$

bahoni inobatga olsak, u holda $D_t^\alpha G(x,t)$ uchun $\forall\,(x,t) \in R \times [\varepsilon,\infty), \forall \varepsilon > 0$ sohada quyidagi baho o'rinli:

$$|D_t^\alpha G(x,t)| = \left| -\frac{a^2}{2\pi} t^{\alpha-1} \int_R e^{-i\xi x} \xi^2 E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) d\xi \right| \le \frac{a^2 t^{\alpha-1}}{2\pi} \int_R \left| e^{-i\xi x} \right| \xi^2 E_\alpha(-a^2\xi^2 t^\alpha) d\xi$$

$$\le \frac{a^2 t^{\alpha-1}}{2\pi} \int_R \xi^2 \frac{\tilde{c} d\xi}{(1+a^2\xi^2 t^\alpha)^2} \le \frac{\tilde{c} t^{-1}}{2\pi} \int_R \frac{d\xi}{1+a^2\xi^2 t^\alpha} \le \frac{\tilde{c} t^{-1}}{2\pi} \int_R \frac{d\xi}{1+a^2\xi^2 t^\alpha} \le c$$

(7) tenglikdan $\forall (x,t) \in R \times [\varepsilon, +\infty)$ da

$$|G_{xx}(x,t)| \le \left| \frac{1}{a^2} D_t^\alpha C \right| \le C$$

bo'lishini ko'rish qiyin emas. 1-lemma to'liq isbotlandi.

1-teorema ning isbotiga to'xtalamiz. Dastlab, $(3)-(4)$ tengliklarning ikkala tomoniga x bo'yicha Furye almashtirish qo'llaymiz. Natijada

$$D_t^\alpha \hat{u} + a^2\xi^2 \hat{u} = 0, 0 < \alpha < 1, \xi \epsilon R, t > 0 \quad (13)$$

$$I_{0+}^{1-\alpha} \hat{u} = \hat{\varphi}(\xi), \xi \in R, \quad (14)$$

tengliklarga ega bo'lamiz, bu yerda

$$\hat{u} = \hat{u}(\xi,t) = \int_R e^{-i\xi x} u(x,t) dx,$$

$$\hat{\varphi} = \hat{\varphi}(\xi) = \int_R e^{-i\xi x} \varphi(x) d\xi.$$

Keyingi bosqichda $\hat{u} = \hat{u}(\xi,t)$ funksiyaning $t > 0$ o'zgaruvchi bo'yicha Laplas almashtirish mavjud deb qaraymiz. So'nra ma'lum formulaga ko'ra,

$$\bar{\hat{u}}(\xi,s) = \frac{\hat{u}(\xi)}{s^\alpha + a^2\xi^2} \quad (15)$$

bo'lishini topamiz. Teskari Laplas almashtirish qo'llab,

$$\hat{u}(\xi,t) = t^{\alpha-1} E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) \cdot \hat{\varphi}(\xi) \quad (16)$$

ekanini ko'rish qiyin emas.

Hosil qilingan (16) tenhlikka ξ o'zgaruvchi bo'yicha teskri Furye almashtirishini qo'llaymiz. Natijada

$$\bar{u}(x,t) = \frac{1}{2\pi} \int_R e^{-i\xi x} t^{\alpha-1} E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) \cdot \hat{\varphi}(\xi) d\xi \quad (17)$$

yoki

$$\hat{\varphi}(\xi) = \int_R e^{i\xi x} \varphi(y) dy$$

funksiyani (17) ga keltirib qo'yish natijasida:

$$u(x,t) = \frac{1}{2\pi} \int_R e^{-i\xi x} t^{\alpha-1} E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha) \int_R e^{i\xi x} \varphi(y) dy\, d\xi$$

$$= \int_R \varphi(y) \left[\frac{1}{2\pi} t^{\alpha-1} \int_R e^{-i\xi(x-y)} E_{\alpha,\alpha}(-a^2\xi^2 t^\alpha)\, d\xi \right] dy$$

$$\bar{u}(x,t) = \int_R G(x-y,t) \varphi(y) dy$$

bo'lishini topdik, bu yerda $G(x,t)$ 1-teoremada aniqlangan. Topilgan yechim (4) shartni qanoatlantirishini tekshirib ko'ramiz. (5) va (7) ga ko'ra,

$$\lim_{t\to 0} I_{0+}^{1-\alpha} \bar{u}(x,t) = \lim_{t\to 0} I_{0+}^{1-\alpha} G(x-y,t) \varphi(y) dy = \int_R \delta(x-y) \varphi(y) dy = \varphi(x)$$

Tenglikdan yechim klassik yechim bo'lganini ko'ramiz. (8) yordamida (5) dan $\forall\, t \ge \varepsilon > 0$, uchun

$$|\bar{u}(x,t)| = \left|\int_R G(x-y,t)\varphi(y)dy\right| \le ||\varphi||_{C_b(R)} \left|\int_R G(x-y,t)dy\right| = \left|\frac{t^{\alpha-1}}{\Gamma(\alpha)}\right| ||\varphi||_{C_b(R)}$$
$$\le \frac{\varepsilon^{\alpha-1}}{\Gamma(\alpha)} ||\varphi||_{C_b(R)}$$

Shunday qilib,

$$|\bar{u}(x,t)| \le C_2 ||\varphi||_{C_b(R)} \tag{18}$$

Undan tashqari, $\varphi \in C^2(R)$ va (9) ga ko‘ra,

$$|D_t^\alpha \bar{u}(x,t)| = \left|\int_R D_t^\alpha G(x-y,t)\varphi(y)dy\right| \le C_3 ||\varphi||_{C_b(R)} \tag{19}$$

Xuddi shunday ko‘rsatish mumkin:

$$|\bar{u}_{xx}(x,t)| = \left|\int_R G_{xx}(x-y,t)\varphi(y)dy\right| \le C_4 ||\varphi||_{C_b(R)}. \tag{20}$$

Demak, (3) tenglamada qatnashayotgan hosilalar mavjud.

2-teorema. Faraz qilaylik, $f(x,t)\epsilon C_b(R\times(0;\infty))$ bo‘lsin. U holda

$$D^\alpha_{\tau+,t} w(x,t,\tau) - a^2 w_{xx}(x,t,\tau) = 0, 0<\alpha<1, t>\tau, x\epsilon R, \tag{21}$$

tenglama

$$I^{1-\alpha} w(x,t,\tau)\,\big|_{t=\tau} = f(x,\tau) \tag{22}$$

Shartni qanoatlantiruvchi yagona yechimi

$$w(x,t,\tau) = \int_R G(x-y,t-\tau) f(y,\tau) dy \tag{23}$$

bu yerda

$$D^\alpha_{\tau+,t} w(x,t,\tau) = \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial t} \int_\tau^t (t-s)^{-\alpha} w(x,s,\tau)\, ds$$

2-teoremaning isboti 1-teorema va 1-lemma dan kelib chiqadi.

Xulosa. Mazkur maqolada vaqt o‘zgaruvchisi bo‘yicha Riman–Liuvill tipidagi kasr tartibli hosilani o‘z ichiga olgan bir o‘lchamli superdiffuziya tenglamasi uchun qo‘yilgan boshlang‘ich–chegaraviy masala atroflicha o‘rganildi. Tadqiqot jarayonida anomal diffuziya jarayonlarini tavsiflovchi ushbu tenglamaning matematik modeli tahlil qilinib, uning yechimini topishda zamonaviy analitik usullardan foydalanildi.

Xususan, masalani yechishda Dyuamel prinsipi, Furye va Laplas almashtirishlari hamda Grin funksiyasi usullari samarali qo‘llanildi. Olingan yechim ikki parametrli Mittag–Leffler funksiyasi orqali aniq ko‘rinishda ifodalandi. Bu esa superdiffuziya tenglamalari uchun yechimlarning xossalarini chuqurroq tahlil qilish imkonini beradi. Shuningdek, yechimning mavjudligi va yagonaligi teoremalar yordamida asoslab berildi hamda Grin funksiyasining muhim xossalari isbotlandi.

Natijalar shuni ko‘rsatadiki, kasr tartibli superdiffuziya tenglamalari klassik diffuziya tenglamalariga nisbatan murakkabroq bo‘lsa-da, ularni analitik usullar yordamida samarali tadqiq etish mumkin. Mazkur ishda olingan natijalar anomal diffuziya jarayonlarini modellashtirishda, xususan, fizik, biologik va muhandislik masalalarida qo‘llanishi mumkin bo‘lib, kelgusida ko‘p o‘lchamli va nolinear superdiffuziya tenglamalarini o‘rganish uchun nazariy asos bo‘lib xizmat qiladi.

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