ifloslanish to‘g‘risidagi ma’lumotlarni ishonchli olish, shuningdek, minimal energiya sarfi bilan monitoring jarayonida yuqori tozalash samaradorligini ta’minlash uchun quyosh energiyasini elektr energiyasiga aylantirish samaradorligini ta’minlashga qaratilgan.

Taklif etilayotgan qurilma quyosh fotoelementlarining sirtini tozalash jarayonini to‘liq avtomatlashtirish hisobiga operator xodimlari ishtirokisiz nafaqat bitta quyosh fotoelementini, balki massivning butun qatori yoki qatorlarini tozalash jarayonining yuqori darajadagi qulayligini va samaradorligini ta’minlaydi, bu esa massivlarni saqlash va ishlatish xarajatlarini kamaytirish, qo‘shimcha energiya ishlab chiqarishga bo‘lgan ehtiyojni maksimal darajada kamaytirish imkonini beradi. anglatadi.

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Vaisova Moxira Davlatboyevna (Urganch davlat universiteti dotsenti; e-mail: v_mohira@mail.ru),
Salayeva Rohila Rustamovna (Urganch davlat universiteti magistranti; e-mail: rokhilasalaeva15@gmail.com)

$\mathbb{C}^2$ FAZODA α -SUBGARMONIK FUNKSIYANING INTEGRAL XOSSASI

Annotatsiya. *Ushbu maqolada $\mathbb{C}^2$ fazoda α-subgarmonik funksiyalar sinfi chiziqli almashtirishlar ostida o‘z xossalarini saqlab qolishi va klassik subgarmonik funksiyalarga o‘xshash muhim o‘rta qiymat xossasiga ega ekani o‘rganilgan.*

Kalit so‘zlar: *α-subgarmonik funksiya, chiziqli almashtirish, differensial forma, invariantlik, o‘rta qiymat xossasi.*

Ваисова Мохира Давлатбоевна (доцент Ургенчского государственного университета),
Салаева Рохила (магистрант Ургенчского государственного университета)

ИНТЕГРАЛЬНОЕ СВОЙСТВО α -СУБГАРМОНИЧЕСКИХ ФУНКЦИЙ В $\mathbb{C}^2$

Аннотация. *В данной статье исследован класс α -субгармонических функций в пространстве $\mathbb{C}^2$, сохраняющих свои свойства при линейных преобразованиях. Доказано, что эти функции обладают важным свойством среднего значения, аналогичным классическим субгармоническим функциям.*

Ключевые слова: *α -субгармоническая функция, линейное преобразование, дифференциальная форма, инвариантность, свойство среднего значения.*

Vaisova Moxira Davlatboyevna (Associate Professor, Urgench State University),
Salayeva Rohila (Master’s Student, Urgench State University)

INTEGRAL PROPERTY OF α -SUBHARMONIC FUNCTIONS IN $\mathbb{C}^2$

Annotation. *This article examines the class of α -subharmonic functions in space $\mathbb{C}^2$, which preserve their properties under linear transformations. It is proven that these functions possess an important mean-value property, similar to classical subharmonic functions.*

Keywords: *α -subharmonic function, linear transformation, differential form, invariance, mean-value property.*

Kirish. Subgarmonik funksiyalar nazariyasi potensiallar nazariyasining asosiy yo‘nalishlaridan biri bo‘lib, ularning xossalari ko‘plab matematik sohalarda, jumladan, kompleks analiz, differensial geometriya va matematik fizika sohalarida muhim rol o‘ynaydi. Subgarmoniklik tushunchasi turli geometrik va analitik strukturalarga bog‘liq holda umumlashtirilishi mumkin. Shunday yondashuvlardan biri bu differensial formalar yordamida berilgan umumlashgan subgarmoniklik, xususan, α -subgarmoniklik tushunchasidir.

Mazkur maqolada α -subgarmonik funksiyaning chiziqli almashtirishlarga nisbatan invariantligi o‘rganiladi. Ya’ni, agar biror funksiya ma’lum bir α -formaga nisbatan subgarmonik bo‘lsa, u holda bu funksiya mos chiziqli almashtirishdan keyin ham yangi formaga nisbatan subgarmonik bo‘lib qoladi. Ush-

bu natija subgarmoniklikning geometrik tabiatini va uning koordinatalar almashtirishiga nisbatan barqarorligini ko‘rsatadi.

Mavzuga oid adabiyotlarning tahlili va tadqiqot metodologiyasi. Keyingi yillarda yaxshi o‘rganilgan subgarmonik va plyurisubgarmonik funksiyalar sinflarning oraliq sinflarini tashkil qiluvchi m — subgarmonik funksiyalar sinfi jadal o‘rganildi va bu sinf uchun potensiallar nazariyasi qurildi va boshqa matematik sohalarga tatbiq qilinmoqda (1,4,5). Bu sinflarni o‘rganish jarayonida differensial formalarni almashtirish orqali yuzaga keladigan yangi bir sinf α -subgarmonik funksiyalarni o‘rganishga zaruriyat tug‘ildi. S.Imomqulov, B.Abdullayev, R.Sharipov, M.Vaisovalarning ishlarida $(n-1,n-1)$ tartibli α -forma orqali $dd^c u \wedge \alpha$ operator aniqlanib, bu operatorning o‘z o‘ziga qo‘shma elliptik operatorligi isbotlandi va α - subgarmonik funksiyalar sinfining ba’zi xossalari isbotlandi (2,3).

Tahlil va natijalar. $D \subset \mathbb{C}^2$ bo‘lsin. Bu sohada birinchi tartibli differensial forma olamiz:

$$\alpha = \frac{i}{2}(\alpha_{11} dz_2 \wedge d\overline{z}_2 + \alpha_{12} dz_2 \wedge d\overline{z}_1 + \alpha_{21} dz_1 \wedge d\overline{z}_2 + \alpha_{22} dz_1 \wedge d\overline{z}_1). \quad (1)$$

α -qat’iy musbat, yopiq forma bo‘lsin: $d\alpha = 0$, α -formaning qat’iy musbatligi deganda istalgan kompakt $G \Subset D$ soha uchun shunday $\varepsilon > 0$ son topiladiki, $\alpha - \varepsilon\beta \geq 0$ munosabat o‘rinli bo‘lishini tushunamiz.

Ma’lumki, differensial formaning musbatlik shartidan uning koeffitsiyentlari $\alpha_{jk} = \overline{\alpha_{kj}}$, $\forall j,k = 1,2$ tenglikni qanoatlantiradi (q. [6]). $dd^c u$ operatorning aniqlanishiga ko‘ra, (q.[4]) $u \in C^2(D)$ uchun $dd^c u = \frac{i}{2}\sum_{j,k=1}^{2} \frac{\partial^2 u}{\partial z_j \partial z_k} dz_j \wedge d\overline{z}_k$ ko‘rinishga ega. Endi yuqorida kiritilgan α differensial forma orqali $dd^c u \wedge \alpha$ operatorni topamiz:

$$dd^c u \wedge \alpha = \left(\frac{i}{2}\right)^2 \left[\alpha_{11} \frac{\partial^2 u}{\partial z_1 \partial \overline{z}_1} + (-1)\alpha_{12} \frac{\partial^2 u}{\partial z_1 \partial \overline{z}_2} + (-1)\alpha_{21} \frac{\partial^2 u}{\partial z_2 \partial \overline{z}_1} + \alpha_{22} \frac{\partial^2 u}{\partial z_2 \partial \overline{z}_2}\right] dz$$

Bu yerda $\frac{i}{2} dz_1 \wedge d\overline{z}_1 = dx_1 \wedge dy_1$, $\frac{i}{2} dz_2 \wedge d\overline{z}_2 = dx_2 \wedge dy_2$ munosabatlardan foydalanib, haqiqiy o‘zgaruvchilarga o‘tamiz va $\Delta_\alpha u$ orqali haqiqiy o‘zgaruvchili differensial operatorni belgilaymiz, belgilashlarni soddalashtirish uchun $y_1 = x_3$; $y_2 = x_4$ deb belgilab olsak, u holda

$$\Delta_\alpha u = \sum_{j,k=1}^{4} a_{j,k} \frac{\partial^2 u}{\partial x_j \partial x_k} \quad (2)$$

ko‘rinishga kelamiz. Bu yerda bu operator koeffitsiyentlaridan tuzilgan

$$A = \begin{pmatrix} \alpha_{11} & -\operatorname{Re}\alpha_{12} & 0 & \operatorname{Im}\alpha_{12} \\ -\operatorname{Re}\alpha_{12} & \alpha_{22} & -\operatorname{Im}\alpha_{12} & 0 \\ 0 & -\operatorname{Im}\alpha_{12} & \alpha_{11} & -\operatorname{Re}\alpha_{12} \\ \operatorname{Im}\alpha_{12} & 0 & -\operatorname{Re}\alpha_{12} & \alpha_{22} \end{pmatrix}$$

matritsasini hosil qilamiz. Bu matritsaning barcha koeffitsiyentlari haqiqiydir. Shu bilan birga, α differensial formaning musbatlik shartidan $\alpha_{11} > 0$, $\alpha_{22} > 0$, $\alpha_{11} \cdot \alpha_{22} - |\alpha_{12}|^2 > 0$ tengsizliklarga

egamiz (q.[6]). Bu matritsa $A = \begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix}$ blok matritsa bo'lib, agar A_1 teskarilanuvchi bo'lsa, $\det A = \det A_1 \det(A_4 - A_3 A_1^{-1} \cdot A_2)$ o'rinli bo'ladi. Bu yerdan $\det A_1 = \begin{vmatrix} \alpha_{11} & -\operatorname{Re}\alpha_{12} \\ -\operatorname{Re}\alpha_{12} & \alpha_{22} \end{vmatrix} = \alpha_{11}\alpha_{22} - \operatorname{Re}^2 \alpha_{12} > 0$ tengsizlikni olamiz.

M.Vaisovaning [3] ishida $\mathbb{C}^n$ fazoda $(n-1, n-1)$ bidarajali qat'iy musbat, yopiq differensial forma uchun $\Delta_\alpha u$ operatorning elliptik operator bo'lishi ko'rsatilgan. B.Abdullayev, S.Imomkulov, R. Sharipovlarning [2] ishida esa quyidagi teorema isbotlangan: agar α - $(n-1, n-1)$ bidarajali qat'iy musbat, yopiq differensial forma bo'lsa, u holda $\Delta_\alpha u$ operator o'z-o'ziga qo'shma elliptik operator bo'-ladi.

1-ta'rif. $D \subset \mathbb{C}^n$ sohada $dd^c u \wedge \alpha = 0$ tenglamani qanoatlantiruvchi $u(z) \in C^2(D)$ funksiyaga α – garmonik funksiya deyiladi.

2-ta'rif. *Agar* $D \subset C^n$ *soha va* $u(z) \in C^2(D)$ *funksiya uchun* $dd^c u \wedge \alpha \geq 0$ *bo'lsa, u holda* α *-subgarmonik funksiya deyiladi. Agar* $D \subset C^n$ *sohada* $u(z) \in L^1_{loc}(D)$ *funksiya uchun*

1) $u(z)$ *funksiya* D *da yuqoridan yarim uzluksiz:* $u(z^0) \geq \overline{\lim_{z \to z_0}} u(z), \forall z^0 \in D$;

2) $dd^c u \wedge \alpha \geq 0$ *operator umumlashgan ma'noda musbat, ya'ni,*

$$\int u(z)\alpha(z) \wedge dd^c \omega(z) \geq 0 \ , \ \forall \omega \in F(D), \ \omega \geq 0.$$

bo'lsa, $u(z)$ *funksiyaga* α *-subgarmonik deyiladi.*

α -subgarmonik funksiyalar sinfi $\alpha - sh(D)$ kabi belgilanadi. Biz $u(z) \equiv -\infty$ funksiya-larni ham $\alpha - sh(D)$ sinfga kiritamiz.

Demak, biz qarayotgan $dd^c u \wedge \alpha$ operator $\mathbb{C}^2$ fazoda ikkinchi tartibli elliptik operatorni ifoda-laydi. [7] ishda bu operatorning fundamental yechimining ko'rinishi topilgan. Bu maqolada $\mathbb{C}^2$ fazoda α -subgarmonik funksiyaning ba'zi xossalarini ko'rib chiqamiz.

1-teorema. Agar $D \subset \mathbb{C}^2$ soha va $u(z) \in \alpha - sh(D), w = T(z) : \mathbb{C}^2 \to \mathbb{C}^2$ xosmas chiziqli akslantirish $(\det T \neq 0)$ bo'lsa, u holda

$$v(w) = u \circ T^{-1}(w)$$

funksiya T chiziqli akslantirish natijasida almashtirilgan α_T formaga nisbatan $D' = T(D)$ sohada α_T-subgarmonik funksiya bo'ladi.

Isbot. Bizga $T : \mathbb{C}^2 \to \mathbb{C}^2$ chiziqli akslantirish

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = T \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} az_1 + bz_2 \\ cz_1 + kz_2 \end{pmatrix}$$

berilgan bo‘lsin, bu yerda $a,b,c,k \in \mathbb{C}$, $\det T = ak - bc \neq 0$. T operatorning $T = \begin{pmatrix} a & b \\ c & k \end{pmatrix}$ matrisasiga teskari T^{-1} operatorni topamiz: $T^{-1} = \dfrac{1}{ak-bc}\begin{pmatrix} k & -b \\ -c & a \end{pmatrix}$.

$w = Tz$ bo‘lsin, u holda $w_1 = az_1 + bz_2$, $w_2 = cz_1 + kz_2$ tengliklardan

$$z_1 = \frac{kw_1 - bw_2}{ak-bc}, \qquad z_2 = \frac{-cw_1 + aw_2}{ak-bc}$$

o‘zgaruvchilarni topamiz. Endi differensiallarni qoida bo‘yicha hisoblasak, quyidagilarga ega bo‘lamiz:

$$dz_1 = \frac{kdw_1 - bdw_2}{ak-bc}, \quad dz_2 = \frac{-cdw_1 + adw_2}{ak-bc}.$$

Shunga o‘xshash qo‘shma differensiallar uchun quyidagi tengliklar o‘rinli:

$$d\overline{z_1} = \frac{\overline{k}d\overline{w_1} - \overline{b}d\overline{w_2}}{\overline{ak}-\overline{bc}}, \; d\overline{z_2} = \frac{-\overline{c}d\overline{w_1} + \overline{a}d\overline{w_2}}{\overline{ak}-\overline{bc}}.$$

Endi topilgan ifodalarni α formaning (1) ko‘rinishiga olib borib qo‘yib, soddalashtirishlardan so‘ng yangi differensial formani olamiz:

$$(T^{-1})^*\alpha = \frac{i}{2}\left(\alpha_{11}{}'dw_2 \wedge d\overline{w_2} + \alpha_{12}{}'dw_2 \wedge d\overline{w_1} + \alpha_{21}{}'dw_1 \wedge d\overline{w_2} + \alpha_{22}{}'dw_1 \wedge d\overline{w_1}\right),$$

bu yerda

$$\alpha_{11}{}' = \frac{\alpha_{11}|a|^2 - \alpha_{12}a\overline{b} - \alpha_{21}b\overline{a} + \alpha_{22}|b|^2}{|ak-bc|^2},$$

$$\alpha_{12}{}' = \frac{-\alpha_{11}a\overline{c} + \alpha_{12}a\overline{k} + \alpha_{21}b\overline{c} - \alpha_{22}b\overline{k}}{|ak-bc|^2} \; \alpha_{21}{}' = \frac{-\alpha_{11}c\overline{a} + \alpha_{12}c\overline{b} + \alpha_{21}k\overline{a} - \alpha_{22}k\overline{b}}{|ak-bc|^2},$$

$$\alpha_{22}{}' = \frac{\alpha_{11}|c|^2 - \alpha_{12}c\overline{k} - \alpha_{21}k\overline{c} + \alpha_{22}|k|^2}{|ak-bc|^2}$$

Bu yangi koeffitsiyentlarni α forma koeffitsiyentlaridan tuzilgan $A = (\alpha_{ij})$ matritsa orqali $A' = \dfrac{(T^{-1})^* A (T^{-1})}{|\det T|^2}$ ko‘rinishda yozish mumkin. Demak, T almashtirish natijasida qat’iy musbat aniqlangan, yopiq α forma yangi qat’iy musbat aniqlangan, yopiq α_T formaga o‘tadi (q.[6]). Agar $u(z)$ funksiya D sohada α - subgarmonik funksiya bo‘lsa, u holda $dd^c u \wedge \alpha \geq 0$ tengsizlik umumlashgan ma’noda bajariladi. $D' = T(D)$ sohada aniqlangan $v(w) = u\left(T^{-1}(w)\right)$ funksiya uchun $dd^c v \wedge \alpha_T = \left(T^{-1}\right)^* \left(dd^c u \wedge \alpha\right)$ tenglik bajariladi. T operator chiziqli va teskarilanuvchi ekanidan $dd^c v \wedge \alpha_T \geq 0$ kelib chiqadi. Demak, v funksiya α_T-formaga nisbatan subgarmonikdir.

1-natija. Agar $w = T(z) : \mathbb{C}^2 \to \mathbb{C}^2$ xosmas chiziqli akslantirish $(\det T \neq 0)$ faqat parallel ko'chirishdan iborat, ya'ni $T(z) = z + b,\ b \in \mathbb{C}$ bo'lsa, u holda almashtirish natijasida α forma o'zgarmaydi va binobarin $v(w) = u \circ T^{-1}(w)$ funksiya yana α -subgarmonik funksiya bo'ladi.

Endi bu tasdiqdan foydalanib, α -subgarmonik funksiyalar uchun muhim ahamiyatga ega bo'lgan o'rta qiymat haqidagi xossani isbotlaymiz.

2-teorema. *Agar $u(z)$ funksiya $D \subset \mathbb{C}^2$ sohada α – subgarmonik bo'lsa, u holda $\forall z^0 \in D$ nuqta va*

$$E(z^0, r) = \left\{ z \in \mathbb{C}^2 : \alpha_{22} \left| z_1 - z_1^0 \right|^2 + \alpha_{21} \left(z_1 - z_1^0 \right) \overline{\left(z_2 - z_2^0 \right)} + \right.$$

$$\left. + \alpha_{12} \left(z_2 - z_2^0 \right) \overline{\left(z_1 - z_1^0 \right)} + \alpha_{11} \left| z_2 - z_2^0 \right|^2 \leq r^2 \right\} \subseteq D$$

psevdoshar uchun

$$u(z^0) \leq \frac{1}{Vol(E(z^0, r))} \int\limits_{E(z^0, r)} u(\tau) dV(\tau)$$

tengsizlik o'rinli bo'ladi. Bu yerda $Vol(E(z^0, r))$ - psevdoshar hajmi va $dV(\tau)$ – hajm elementi.

Isbot: (1) forma koeffitsiyentlaridan tuzilgan $A = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$, $\alpha_{21} = \overline{\alpha_{12}}$ matritsa musbat aniqlangan ermit matritsasidir. Shuning uchun shunday U unitar almashtirish mavjudki, $A = U^* \Lambda U$, $\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$, $\lambda_1, \lambda_2 > 0$ munosabatlar o'rinli. Quyidagi $w = T(z - z^0) = \Lambda^{\frac{1}{2}} U (z - z^0)$ chiziqli almashtirishni qaraymiz:

$$\begin{cases} w_1 = \sqrt{\lambda_1} \left(U_{11}(z_1 - z_1^0) + U_{12}(z_2 - z_2^0) \right) \\ w_2 = \sqrt{\lambda_2} \left(U_{21}(z_1 - z_1^0) + U_{22}(z_2 - z_2^0) \right) \end{cases}.$$

Bu akslantirish natijasida, $E\left(z^0, r\right)$ psevdoshar odatdagi Evklid fazosidagi sharga akslanadi:

$$E\left(z^0, r\right) \to B(0, r) = \left\{ w \in \mathbb{C}^2 : \left| w_1 \right|^2 + \left| w_2 \right|^2 \leq r^2 \right\}$$

Bu akslantirish yakobiani $J = \frac{1}{\lambda_1 \lambda_2} = \frac{1}{\det A}$ ga teng bo'ladi. $u(z)$ funksiyani yangi koordinatalarda ifodalasak, bu funksiya

$$\tilde{u}(w) = u\left(z^0 + U^* \Lambda^{-\frac{1}{2}} w \right)$$

ko'rinishga o'tadi. $u(z)$ funksiya $D \subset \mathbb{C}^2$ sohada α -subgarmonik funksiya ekanligidan 1-teoremaga ko'ra chiziqli xosmas almashtirish natijasida hosil bo'lgan $\tilde{u}(w)$ funksiya D sohaning aksi bo'l-

gan D' sohada odatdagi subgarmonik funksiyani ifodalaydi. Subgarmonik funksiyalar nazariyasidan ma'lumki (5), bu funksiya uchun o'rta qiymat haqidagi teorema o'rinli:

$$\tilde{u}(0) \le \frac{1}{Vol\big(B(0,r)\big)} \int\limits_{B(0,r)} \tilde{u}(w) dV(w),\ Vol\big(B(0,r)\big) = \frac{\pi^2 r^4}{2}$$

Endi bu integralni dastlabki o'zgaruvchilar orqali ifodalasak,

$$\tilde{u}(w) = u(z^0 + T^{-1}w),\ \tilde{u}(0) = u(z^0),$$

$Vol(B(0,r)) = \left|\det T\right|^2 Vol(E(z^0,r))$ tengliklardan

$$\frac{1}{Vol\big(B(0,r)\big)} \int\limits_{B(0,r)} \tilde{u}(w) dV(w) = \frac{1}{Vol\big(B(0,r)\big)} \int\limits_{E(z^0,r)} u(z) \left|\det T\right|^2 dV(z) =$$

$$= \frac{1}{Vol\big(E(z^0,r)\big)} \int\limits_{E(z^0,r)} u(z) dV(z)$$

ifodani hosil qilamiz. Bu yerdan esa

$$u(z^0) \le \frac{1}{Vol(E(z^0,r))} \int\limits_{E(z^0,r)} u(z) dV(z)$$

tengsizlikni olamiz, bu yerda $E(z^0,r)$ psevdoshar hajmi. Shu bilan birga, $Vol(E(z^0,r)) = \dfrac{\pi^2 r^4}{2\left|\det T\right|^2} = \dfrac{\pi^2 r^4}{2 \det A}$ ga tengligini ham topishimiz mumkin.

Xulosa. Ushbu maqoladagi tadqiqot shuni ko'rsatadiki, α -subgarmonik funksiyalar sinfi chiziqli almashtirishlar ostida o'z xossalarini saqlab qoladi, bu ularning kompleks analiz va matematik fizikadagi ahamiyatini yanada oshiradi. 1-teorema α -subgarmoniklikning geometrik barqarorligini tasdiqlasa, 2-teorema esa klassik subgarmonik funksiyalarga o'xshash muhim o'rta qiymat xossasini isbotlaydi. Bu natijalar kompleks fazolarda potensiallar va differensial operatorlarni o'rganish uchun yangi yo'nalishlarni ochadi. Keyingi tadqiqotlar kengroq funksiyalar sinflari va ko'pxilliklar uchun olingan natijalarni umumlashtirishga qaratilishi mumkin.

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