(4). Nagel A., Rudin W. Moebius-invariant function spaces on balls and spheres. Duke Math. J. 1976, V. 43, p. 841–865.

(5). Косбергенов С. О граничной теореме Морера для шара и поликруга. Комплексный анализ и математическая физика. Красноярск, КрасГУ, 1998, с. 59–65.

(6). Косбергенов С., Кытманов А.М., Мысливец С.Г. О граничной теореме Морера для классических областей. "Сибирский математический журнал", 1999, т. 40, № 3, с. 595–604.

(7). Косбергенов С. О многомерной граничной теореме Мореры для матричного шара. Известия вузов. Математика, 2001, № 4 (467), с. 28–32.

(8). Худайберганов Г., Кытманов А.М., Шаимкулов Б.А. Комплексный анализ в матричных областях. Красноярск, Сибирский федеральный университет, 2011, 290 с.

(9). Хуа Локен. Гармонический анализ функций многих комплексных переменных в классических областях. Москва, ИЛ, 1959.

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FRAKTAL-KASR TARTIBLI DIFFERENSIAL TENGLAMALAR YORDAMIDA KORRUPSIYA DINAMIKASINI MATEMATIK TADQIQ QILISH

Annotatsiya. *Maqolada korrupsiya jarayonini xotira effekti va ijtimoiy muhitdagi notekis ta'sirlarni hisobga olgan holda ifodalovchi fraktal-kasr tartibli matematik model taklif qilinadi. Modelda korrupsiya intensivligi, institutsional nazorat va jamoatchilik shaffofligi o'zaro bog'langan o'zgaruvchilar sifatida qaraladi. Natijalar korrupsiyaga qarshi siyosatda nazorat, shaffoflik va xotira parametrlarining rolini izohlash imkonini beradi.*

Kalit so'zlar: *fraktal-kasr hosila, korrupsiya dinamikasi, matematik model, Caputo operatori, barqarorlik.*

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МАТЕМАТИЧЕСКОЕ ИССЛЕДОВАНИЕ ДИНАМИКИ КОРРУПЦИИ С ПОМОЩЬЮ ФРАКТАЛЬНО-ДРОБНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

Аннотация. *В статье построена фрактально-дробная математическая модель динамики коррупции с учетом эффекта памяти и неоднородности социальной среды. Коррупционная активность, институциональный контроль и общественная прозрачность рассматриваются как взаимосвязанные переменные. Полученные результаты показывают стабилизирующее влияние контроля и прозрачности.*

Ключевые слова: *фрактально-дробная производная, динамика коррупции, математическая модель, оператор Капуто, устойчивость.*

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MATHEMATICAL INVESTIGATION OF CORRUPTION DYNAMICS USING FRACTAL-FRACTIONAL DIFFERENTIAL EQUATIONS

Annotation. *This article develops a fractal-fractional differential model for corruption dynamics under memory effects and heterogeneous social influence. Corruption activity, institutional control and public transparency are considered as mutually connected dynamic variables. The methodology is based on a Caputo-type fractal-fractional operator, positivity of solutions, equilibrium analysis and a local stability condition. The proposed model is theoretical and can be extended to empirical data after parameter identification.*

Keywords: *fractal-fractional derivative, corruption dynamics, mathematical model, Caputo operator, stability, transparency.*

Introduction. Corruption is a complex socio-economic process in which incentives, institutional weakness, delayed punishment, informal networks and public tolerance interact over time. In many cases the present level of corruption is influenced not only by current control measures, but also by previous administrative culture, accumulated informal experience and delayed legal reaction. Therefore, ordinary differential equations with only local time dependence may be insufficient.

Fractional differential equations are useful for processes with memory. The additional fractal component reflects irregular social interaction, heterogeneous administrative structure and non-uniform transmission of corrupt behavior. The purpose of this article is to construct a compact theoretical model of corruption dynamics by means of fractal-fractional differential equations and to analyze positivity, equilibrium and stability conditions.

Literature review. Fractional calculus is widely used to describe hereditary behavior, anomalous diffusion and nonlocal time dynamics [1;78–83 p.], [2;15–22 p.]. The Caputo derivative is convenient for applied modelling because it allows classical initial conditions. Stability of fractional systems is usually connected with the Mittag-Leffler function, which generalizes the exponential function [3;92–100 p.].

Mathematical models of corruption are often based on epidemic-type systems, game theory and optimal control. In such models corrupt behavior may be interpreted as a transmissible social state. The present article differs from simple integer-order models by introducing a fractal-fractional operator and by treating institutional control and public transparency as dynamic feedback variables.

Research Methodology. Let $C(t)$ denote the normalized intensity of corruption activity, $I(t)$ the level of institutional control, and $P(t)$ the level of public transparency at time $t \geq 0$. All variables are nonnegative. For $0 < \alpha \leq 1$ and $0 < \beta \leq 1$, we use the Caputo-type fractal-fractional derivative

$$D_{FF}^{\alpha,\beta} x(t) = \frac{t^{1-\beta}}{\beta \Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} x'(s)\, ds.$$

Here α controls memory depth and β reflects the irregularity of interaction time. The proposed model is

$$D_{FF}^{\alpha,\beta} C = \lambda C\left(1 - \frac{C}{K}\right) - (\mu + \gamma I + \rho P)C.$$

$$D_{FF}^{\alpha,\beta} I = s + \frac{\eta C}{m + C} - dI.$$

$$D_{FF}^{\alpha,\beta} P = q + \frac{\theta C}{m + C} - \xi P.$$

with initial conditions

$$C(0) = C_0 \geq 0, \qquad I(0) = I_0 \geq 0, \qquad P(0) = P_0 \geq 0.$$

The parameters have the following meanings: λ is the growth intensity of corruption, K is the saturation level, μ is the natural removal rate, γ and ρ are the effects of control and transparency, s and q are baseline inputs, η and θ are reaction coefficients, d and ξ are decay rates, and m is a half-saturation coefficient [4].

Analysis and results. The right-hand side of the system is locally Lipschitz in the nonnegative domain. Hence, by the fixed-point argument for fractional Volterra equations, the model has a unique local solution. Its integral form is

$$X(t) = X(0) + \frac{\beta}{\Gamma(\alpha)} \int_0^t \tau^{\beta-1} (t-\tau)^{\alpha-1} F\big(X(\tau)\big)\, d\tau,$$

where $X(t) = \big(C(t), I(t), P(t)\big)^T$. This expression demonstrates that the current state depends on the previous trajectory of the system.

The nonnegative region is invariant. On the boundary $C = 0$ the first equation gives zero; on $I = 0$ the second equation is nonnegative; and on $P = 0$ the third equation is nonnegative. Thus, nonnegative initial data generate nonnegative solutions.

The corruption-free equilibrium is

$$E_0 = \left(0, \frac{s}{d}, \frac{q}{\xi}\right).$$

At this equilibrium, the effective growth rate of corruption is

$$\sigma_0 = \lambda - \mu - \gamma \frac{s}{d} - \rho \frac{q}{\xi}.$$

Therefore, the corruption reproduction indicator can be written as

$$\mathcal{R}_c = \frac{\lambda}{\mu + \gamma s/d + \rho q/\xi}.$$

If $\mathcal{R}_c < 1$, then the corruption-free equilibrium is locally asymptotically stable in the fractional sense. If $\mathcal{R}_c > 1$, corruption may persist and an endemic corruption level can appear. The formula shows that increasing baseline control s, transparency q, and their efficiencies γ, ρ reduces the corruption reproduction indicator.

The fractional order α explains why anti-corruption measures may have delayed effects: the system remembers previous weak-control periods. The fractal parameter β changes the effective time scale and may describe irregular reform implementation, unequal institutional capacity or fragmented information flow. For qualitative scenario analysis one may compare three regimes. In the first regime, baseline institutional control and transparency are weak; then the denominator of $\mathcal{R}_c$ is small and corruption can grow even from a low initial level. In the second regime, control increases but transparency remains weak; in this case the model predicts reduction of corruption, but the decrease may be slow because public pressure does not support institutional measures. In the third regime, both control and transparency are strengthened; then the threshold indicator decreases most effectively and the corruption-free equilibrium becomes more robust. This interpretation is important because it shows that institutional policy and public openness should not be treated as isolated factors [5].

The model also explains the difference between short-term and long-term effects. A reform may increase $I(t)$ quickly, but if $\alpha < 1$, the trajectory of $C(t)$ still contains the memory of previous weak-control periods. Therefore, the immediate reduction of corruption may be limited, while the long-term tendency becomes stable only after persistent action. This feature is one of the main advantages of the fractal-fractional approach compared with an ordinary differential equation model.

A further qualitative result follows from the threshold formula. If the state policy increases only the punishment term μ, but does not improve institutional control $I(t)$ or transparency $P(t)$, the model may still be unstable when λ is large. This means that punitive measures alone are not enough in a system where informal incentives reproduce corruption. A stable policy must increase the permanent denominator of $\mathcal{R}_c$, that is, baseline monitoring, detection probability, administrative discipline and social transparency.

From the mathematical point of view, the threshold $\mathcal{R}_c$ plays the same role as a reproduction number in epidemic-type models. However, its interpretation is social rather than biological. If $\mathcal{R}_c < 1$, one corruption episode produces, on average, less than one effective continuation of corrupt practice in the institutional environment. If $\mathcal{R}_c > 1$, informal incentives are strong enough to maintain the process. This interpretation makes the model useful for comparative scenarios: before estimating exact numbers, one can determine which group of parameters should be strengthened to move the system toward the stable region [6].

The proposed system may also be used for numerical simulation. For example, the Adams-Bashforth-Moulton predictor-corrector method for fractional equations can be applied to the integral form of the model. In such simulations, a decrease in α usually produces slower convergence to equilibrium, because the memory kernel gives larger weight to the historical trajectory. A decrease in β changes the effective time deformation and may represent irregular institutional response [7].

It should be emphasized that the variables of the model are normalized indices. They do not accuse any concrete organization or person, and they should be interpreted only as abstract mathematical quantities. To apply the model in practice, the researcher must define measurable proxies, such as audit frequency, number of detected cases, transparency indicators or survey-based trust indices. After this step, the parameters can be estimated by least squares, Bayesian methods or other identification procedures.

Conclusion/Recommendations. The article proposed a fractal-fractional differential model for corruption dynamics. The model combines corruption activity, institutional control and public transparency in one system. The main results are the positivity of solutions, the corruption-free equilibrium and the threshold condition $\mathcal{R}_c < 1$ for local stability.

The analysis confirms that institutional control and public transparency are stabilizing factors, whereas a high natural growth rate of corrupt behavior increases persistence risk. For future research, it is recommended to estimate parameters from empirical indicators, compare the model with an integer-order system and construct numerical schemes for scenario analysis.

References:

[1]. Podlubny I. Fractional Differential Equations. San Diego: Academic Press, 1999, 340 p.

[2]. Kilbas A.A., Srivastava H.M., Trujillo J.J. Theory and Applications of Fractional Differential Equations. Amsterdam: Elsevier, 2006, 523 p.

[3]. Diethelm K. The Analysis of Fractional Differential Equations. Berlin: Springer, 2010, 247 p.

[4]. Rose-Ackerman S., Palifka B.J. Corruption and Government: Causes, Consequences, and Reform. Cambridge: Cambridge University Press, 2016, 618 p.

[5]. N.M.Jabborov, S.E.Eshdavlatova, S.G‘.To‘ychiyev. “Влияние некоторых коэффициентов на уровень коррупции”. “Yosh olimlar axborotnomasi”, 4(4), 2024.

[6]. N.M.Jabborov, S.E.Eshdavlatova, B.N.Mirzoodilov “Yashirin iqtisodiyotni dinamik tizimlar va kasr tartibli differesial tenglamalar yordamida tadqiq qilish”. “Innovations in Science and Technologies”, ilmiy-elektron jurnali, ISSN: 3030-3451, 2-son, 2025.

[7]. N.M.Jabborov, B.E.Husanov, S.E.Eshdavlatova. “A Mathematical Model of Corruption Dynamics with Prevention Parameters”. ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2025, vol. 46, № 7, pp. Pleiades Publishing, Ltd., 2025. 3595-3604 pages.