3.4. **Интегрирование по тору. Задача.** Вычислить объём тора с большим радиусом R и малым радиусом r.

Решение. В тороидальных координатах или с помощью теоремы Паппа объём равен $2\pi^2 Rr^2$ (R>r>0). С использованием ParametricRegion:

```
torusParams = {(R + r Cos[v]) Cos[u], (R + r Cos[v]) Sin[u],
r Sin[v]};
Rparam = ParametricRegion[torusParams, {{u, 0, 2 Pi}, {v, 0,
2 Pi}}];
```

Листинг 14. Вычисление объёма тора.

3.5. Визуализация области проверки.

```
coordinates = {r Cos[theta], r Sin[theta], z};
R = ParametricRegion[coordinates,
{{r, r_min, r_max}, {theta, theta_min, theta_max}, {z, z_min,
z_max}}];
RegionPlot3D[R, PlotPoints -> 50, Axes -> True]
```

Листинг 15. Визуализация области интегрирования.

Эта простая проверка выявляет ошибки в задании границ, которые иначе испортили бы результаты.

Заключение. Mathematica предоставляет всеобъемлющую среду для тройного интегрирования, охватывающую символьное вычисление, адаптивные численные методы и геометрическую визуализацию. Ключ к эффективному использованию лежит в стратегическом суждении: выбор символьного интегрирования, когда подынтегральная функция и область допускают замкнутые формы; применение численных методов в противном случае; и всегда рассмотрение преобразований координат и перестановок порядка интегрирования как инструментов решения задач. Mathematica служит не заменой аналитическому мышлению, а его усилителем – автоматизируя рутинные вычисления, оставляя стратегические решения о выборе метода, системы координат и оценке погрешностей человеческому суждению. Представленные примеры показывают, что наиболее эффективные вычисления возникают при продуманном сочетании математической интуиции и вычислительных возможностей.

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Sharipova Shohista Bakbergan qizi (Teacher at Urgench State University named after Abu Rayhan Beruni).
Olimova Oyshajon Allabergan qizi (Student at Urgench State University named after Abu Rayhan Beruni);
e-mail: shshb1990@gmail.com)

METHODOLOGY FOR ANALYZING THE EQUILIBRIUM OF A SYSTEM OF INTERSECTING FORCES AT A POINT IN ENGINEERING CALCULATIONS

Annotation. *This paper presents a comprehensive analysis of one of the fundamental problems in statics–the equilibrium of a system of intersecting forces at a point. The study focuses on the theoretical foundations and practical implementation of the analytical method based on projecting forces onto coordinate axes. Particular emphasis is placed on the accuracy, efficiency, and applicability of this method*

in engineering calculations. Several illustrative problems are analyzed to demonstrate the effectiveness of the proposed approach. The results confirm that the analytical method provides a reliable and systematic framework for determining unknown forces and ensuring equilibrium conditions in various mechanical systems.

Key words: *statics, system of intersecting forces, equilibrium, force projection, analytical method, resultant force.*

MUHANDISLIK HISOB-KITOBLARIDA BIR NUQTADA KESISHUVCHI KUCHLAR TIZIMINING MUVOZANATINI TAHLIL QILISH METODOLOGIYASI

Annotatsiya. *Ushbu maqolada statikaning asosiy masalalaridan biri bir nuqtada kesishuvchi kuchlar sistemasining muvozanati har tomonlama tahlil qilinadi. Tadqiqot kuchlarni koordinata o'qlari bo'yicha proyeksiyalashga asoslangan analitik usulning nazariy asoslari va amaliy qo'llanilishiga qaratilgan. Mazkur usulning aniqligi, samaradorligi va muhandislik hisoblaridagi qo'llanilish imkoniyatlariga alohida e'tibor berilgan. Taklif etilgan yondashuvning samaradorligini ko'rsatish maqsadida bir nechta namunaviy masalalar tahlil qilingan. Olingan natijalar analitik usul turli mexanik tizimlarda noma'lum kuchlarni aniqlash va muvozanat shartlarini ta'minlash uchun ishonchli va tizimli vosita ekanligini tasdiqlaydi.*

Kalit so'zlar*: statika, kesishuvchi kuchlar sistemasi, muvozanat, kuchlarning proyeksiyasi, analitik usul, teng ta'sir etuvchi kuch.*

МЕТОДОЛОГИЯ АНАЛИЗА РАВНОВЕСИЯ СИСТЕМЫ СХОДЯЩИХСЯ СИЛ В ТОЧКЕ В ИНЖЕНЕРНЫХ РАСЧЕТАХ

Аннотация. *В данной работе представлен всесторонний анализ одной из фундаментальных задач статики– равновесия системы сходящихся сил в одной точке. Исследование сосредоточено на теоретических основах и практической реализации аналитического метода, основанного на проекциях сил на координатные оси. Особое внимание уделяется точности, эффективности и применимости данного метода в инженерных расчетах. Рассмотрены типовые задачи, демонстрирующие эффективность предложенного подхода. Полученные результаты подтверждают, что аналитический метод является надежным и систематическим инструментом для определения неизвестных сил и обеспечения условий равновесия в различных механических системах.*

Ключевые слова*: статика, система сходящихся сил, равновесие, проекция сил, аналитический метод, равнодействующая сила.*

Introduction. In engineering and technology, the study of forces acting on bodies is of great importance. In particular, a system of forces intersecting at a point is one of the fundamental concepts of mechanics [1–3]. Such systems are widely used to determine the equilibrium of bodies, evaluate the strength of structures, and solve various mechanical problems [4–6]. A system of intersecting forces refers to a situation in which the lines of action of all forces meet at a common point. By analyzing such a system, it is possible to determine the resultant force, its direction, and its overall effect. This plays an essential role in practical engineering calculations [7]. This article examines the system of intersecting forces at a point, its main properties, and its equilibrium conditions.

Research methodology. The laws of statics, particularly those related to force equilibrium, are based on the fundamental works of scientists such as I.Newton and J.L.Lagrange. This study employs an analytical method based on projecting forces onto coordinate axes [8]. Compared to graphical and geometric methods, the analytical method provides more accurate and systematic results.

The analysis shows that, to determine the resultant force, it is necessary to calculate the sum of the projections of all forces on the coordinate axes [9].

$$\sum F_{iy} = 0;\ \ R_x = \sum F_{ix}$$

$$R_y = \sum F_{iy}$$

For example, when analyzing forces acting on the base of a vertical column, the horizontal component of a force can be determined using trigonometric relations. $R_x = F_1 \cdot \cos\alpha + F_2 + F_3 \cdot \cos\alpha$

In equilibrium conditions, the sum of force projections along each axis must be equal to zero.

$$\sum F_{ix} = 0; \qquad \sum F_{iy} = 0;$$

Research methodology. I. Vertical column (wire and rod system). Three forces act at point A of the structure. $F_1 = F_2 = F_3 = 10N$. These forces lie in the same plane and are applied at specific angles. $\alpha = 60°$ The task is to determine the horizontal projection of the resultant force. $sin\ 60° \approx 0,866$ (Fig 1).

II. Rigid Body Equilibrium. Four forces act at point O of a rigid body: $F_1 = 1N$, $F_2 = 2N$, $F_3 = 3N$, $F_4 = 4N$. If these forces O_x and O_y against the arrows $\alpha = 30^0$, $\beta = 30^0$, $\gamma = 30^0$, given the directions of these forces and the corresponding angles, the task is to determine the projection of the resultant force on a specified axis (Fig 2).

III. Blocks and threads system. A system of loads is suspended by ropes over pulleys and is in equilibrium. Given the angles $G_2 = 55N$ and $\alpha = 75°, \beta = 60°$ known forces, the task is to determine the magnitude of the unknown force (Fig 3).

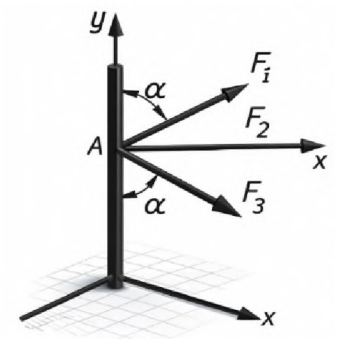


Figure 1.

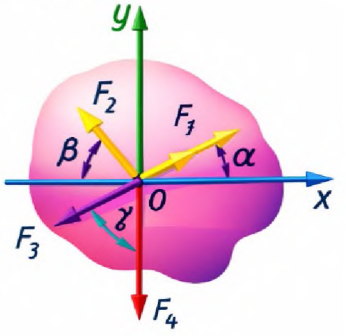


Figure 2.

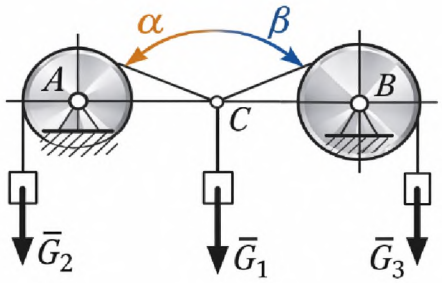


Figure 3.

Analysis and results. I. Solution. According to the laws of mechanics, the combined effect of several forces is represented by their vector sum. First, the horizontal projection of each force is determined.

$F_x = F \cdot \cos\alpha$,

From this,

$F_{1x} = F_1 \cdot \cos 30° = 10 \cdot 0,866 = 8,66N$

$F_{2x} = F_2 \cdot \cos 0° = 10 \cdot 1 = 10N$

$F_{3x} = F_3 \cdot \cos 30° = 10 \cdot 0,866 = 8,66N$

The horizontal component of the resultant force is then found by summing the projections of all external forces on the chosen axis. $R_x = \sum F_{ix} = F_{1x} + F_{2x} + F_{3x} = 8,66 + 10 + 8,66 = 27,32N$

Answer: (as obtained from calculations) $R_x = 27,32N$

II. Solution. The projection of a system of forces on an axis is equal to the sum of the projections of the individual forces on that axis. Based on the diagram, the force components are calculated, and their projections onto the axis are determined.

$F_{iy} = F_i \cos\theta_i$

$F_{1y} = F_1 = 1N$

$F_{2y} = F_2 \sin\alpha = 2 \cdot \sin 30° = 2 \cdot 0,5 = 1N$

$F_{3y} = -F_3 \cos\beta = -3 \cdot \cos 45° = -3 \cdot 0,707 \approx -2,12N$

$F_{4y} = -F_4 \sin\gamma = -4 \cdot \sin 60° = -4 \cdot 0,866 \approx -3,46N$

If a force acts in the same direction as the axis, its projection is taken as positive. The total projection is then obtained by summing all individual projections.

$$R_y = \sum F_{iy} = F_{1y} + F_{2y} + F_{3y} + F_{4y}$$

General projection:

$$R_y = 1 + 1 - 2,12 - 3,46 = -3,58N$$

Answer: (as obtained from calculations) $R_y = -3,58N$

III. Solution. Three forces act at the node: a tension force directed to the left, a tension force directed to the right, and a downward gravitational force. Since the system is in equilibrium, the sum of the projections of all forces along each axis must be equal to zero. By resolving the forces into horizontal and vertical components and applying equilibrium conditions, the unknown force is determined using trigonometric relationships.

$$-T_A \cdot \sin\alpha + T_B \cdot \sin\beta = 0, \quad T_A = G_2 \qquad T_B = G_3$$

$$G_2 \cdot \sin\alpha = G_3 \cdot \sin\beta \qquad G_3 = \frac{G_2 \sin\alpha}{\sin\beta}$$

$$G_3 = \frac{G_2 \sin\alpha}{\sin\beta} = \frac{55 * \sin(75^0)}{\sin(60^0)} \approx 61.34N$$

Answer: (as obtained from calculations) $G_3 \approx 61.34N$

In this problem, the equilibrium of a system of forces transmitted through pulleys is analyzed. The forces acting at the node are resolved into horizontal and vertical components, and equilibrium conditions are applied. The results show that the equilibrium depends directly on the directions and angles of the ropes. By correctly analyzing these parameters, unknown quantities can be accurately determined.

Conclusion. Three main methods are used to analyze a system of intersecting forces at a point:

•the geometric method;

•the graphical method;

•the analytical method.

In practice, the most effective approach is the analytical method, which is based on resolving forces into components using trigonometric functions. Such systems are widely encountered in building structures, mechanical devices, lifting systems, and various engineering applications. A clear understanding of systems of intersecting forces provides a solid foundation for studying more advanced topics in mechanics. It helps explain the interaction of forces, the laws of equilibrium, and the nature of physical processes in a deeper and more systematic way.

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Abdulkhayev Abrorbek Abdullokhon ugli (Doctor of Philosophy (PhD) in Physical and Mathematical Sciences; [1]Namangan State Technical University; E-mail: abdulxayev2308@gmail.com),

Mamatkarimov Odiljon Okhundedaevich (Doctor of Physical and Mathematical Sciences, Professor, Namangan State Technical University),

Quchqarov Behzod Hoshimjonovich (Doctor of Philosophy (PhD) in Physical and Mathematical Sciences, Associate Professor, Institute of Impulse Medicine)

INFLUENCE OF AN INTERFACIAL SiO_2 LAYER ON CHARGE TRAPPING AND LEAKAGE CURRENT IN SEMICONDUCTOR–DIELECTRIC STRUCTURES

Annotation. *The continuous scaling of semiconductor devices requires the use of materials with high dielectric permittivity. In this regard, titanium dioxide (TiO_2), due to its high dielectric constant, is considered a promising material for metal–dielectric–semiconductor (MDS) structures. However, the direct contact of the TiO_2 layer with the silicon substrate leads to an increase in the density of interface states, an increase in leakage current, and the appearance of hysteresis in capacitance–voltage (C–V) characteristics. In this work, the improvement of interface quality by introducing a thin silicon dioxide (SiO_2) interfacial layer between the silicon substrate and the TiO_2 dielectric layer is considered. Within the*