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OSILGAN MASSALI TO'G'RI TO'RBURCHAKLI QOVUSHOQ-ELASTIK ORTOTROP PLASTINKANING XOS TEBRANISHLARI

Annotatsiya. *Osilgan massalari bo'lgan to'g'ri to'rburchakli qovushoq-elastik ortotrop plastinkaning xos tebranishlari qaralgan. Energetik usuldan foydalanib harakat tenglamalari olingan, chegaraviy shartlar shakllantirilgan. Qo'yilgan masalani yechish uchun Reley-Rits usuli qo'llangan. Sonli natijalar shuni ko'rsatadiki, massalar nisbatining o'zgarishi chastotalar haqiqiy qismlarining o'zgarishiga sabab bo'lar ekan. Materiallarning qovushoqlik xossalarini hisobga olish ko'chish va kuchlanishlarni 10–15% ga kamaytirishga imkon beradi.*

Kalit so'zlar: *ortotrop plastinka, osilgan massa, xos tebranishlar, chastota, qovushqoqlik.*

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СОБСТВЕННЫЕ КОЛЕБАНИЯ ПОДВЕСНОЙ МАССЫ УПРУГОЙ ОРТОТРОПНОЙ ПЛАСТИНЫ С ПРЯМОУГОЛЬНЫМ ШАРНИРОМ

Аннотация. *Рассматриваются собственные колебания прямоугольной упругой ортотропной пластины с подвешенными массами. С помощью энергетического метода были выведены уравнения движения и сформулированы граничные условия. Для решения задачи использовался метод Рэлея-Ритца. Численные результаты показывают, что изменение соотношения масс приводит к изменению действительных частей частот. Учет вязкостных свойств материалов позволяет снизить перемещения и напряжения на 10–15%.*

Ключевые слова: *ортотропная пластина, подвешенная масса, собственные колебания, частота, вязкость.*

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NATURAL VIBRATIONS OF A SUSPENDED MASS ELASTIC ORTHOTROPIC PLATE WITH A RECTANGULAR JOINT

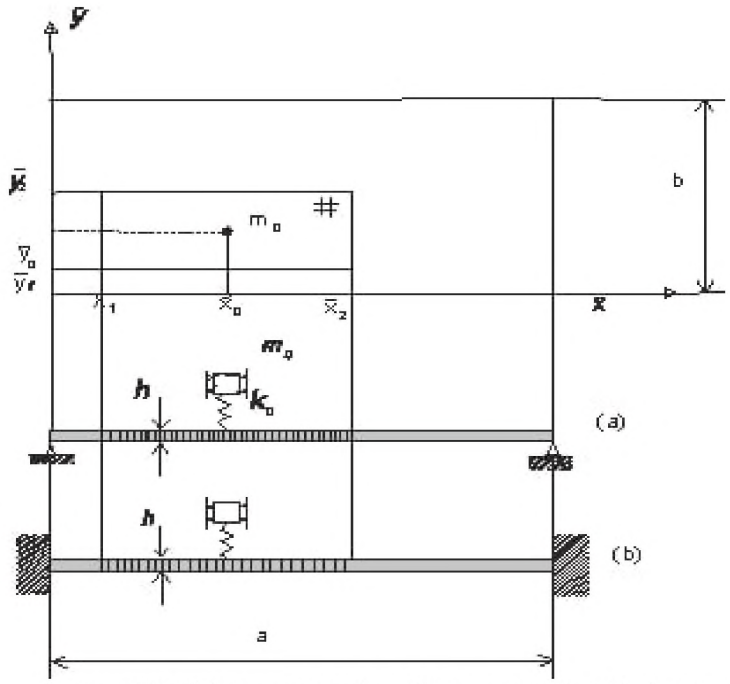
Annotation. *The natural vibrations of a rectangular viscous elastic orthotropic plate with suspended masses are considered. The equations of motion are obtained using the energetic method, and boundary conditions are formulated. The Rayleigh-Ritz method is used to solve the problem. Numerical results show that a change in the mass ratio causes a change in the real parts of the frequencies. Taking into account the viscous properties of materials allows reducing displacements and stresses by 10–15%.*

Keywords: *orthotropic plate, suspended mass, natural vibrations, frequency, viscosity.*

1. Kirish. Harakatlanuvchi obyektlarga (samolyotlar, kemalar, o'ziyurar mashinalar, avtomobillar va boshqalar) joylashtirilgan zamonaviy radiomexanik tizimlar va navigasiya uskunalarining turg'unligini ta'minlovchi omillar murakkab qobiq sistemasidan iborat bo'ladi [1;2-b.]. Harakatlanuvchi jismlarning konstruktiv elementlariga ta'sir etuvchi omillar yuzaga keladigan tebranish va zarba yuklari, akustik effektlar va boshqalar kiradi. Aerodinamik kuchlar harakatlanuvchi apparatlarning elementlari va qismlarining vibrasion tebranishlarini oshiradi. Bunday tuzilmalarni loyihalashda strukturaning xavfsizlik chegaralarini aniqlash, rezonans chastotalarini va strukturaning detallariga ta'sir qiluvchi faktorlarni hisoblash uchun dinamik hisob-kitoblarni amalga oshirish qo'yidagi ishlarga keltirilgan [3;4-b.]. Shunga o'xshash muammolar elastiklik nazariyasining uch o'lchovli dinamik chegaraviy muammolari bilan bog'liq [5]. Radio texnik apparatlarni hisoblash murakkablashadi, chunki bunday tuzilmalar juda ko'p kichik qismlarga ega bo'lgan murakkab shaklga ega elementlardan iborat.

2. Metodlar. 2.1. Masalaning qo'yilishi va yechish usullari. O'rnatilgan (yoki yopishtirilgan) va osilgan massasi bo'lgan to'g'ri to'rtburchakli plastinkaning tebranishlari massalasini ko'ramiz. Plastinkaga massalar prujinkalar orqali osilgan. Plastinka va prujinkalar qovushoq-elastiklik xususiyatiga ega. Bu plastinka va unga osilgan moddiy nuqtalardan tashkil topgan mexanik sistemaning erkin va majburiy tebranishlarini ko'ramiz. Mexanik sistema dissipativ bir jinsli yoki bir jinsli emas, deb o'rganiladi.

Faraz qilamiz, 1-rasmda tasvirlangan hisob sxemasi berilgan bo'lsin.



1-rasm. Plastinka va unga o‘rnatilgan massaning hisob sxemasi.

Taqribiy analitik yechimni olish uchun plastinkaning klassik nazariyasidan energiya funksionalini tebranayotgan plastinka uchun qurish mumkin (1-rasm).

$$J(W,Z)=\iint_{\Omega_s}\left[\bar{D}_1W_{xx}^2+2\mu_2\bar{D}_1W_{xx}W_{yy}+\bar{D}_2W_{yy}^2+4\bar{D}_kW_{xy}^2\right]dxdy-$$

$$-\frac{1}{2}\rho\omega^2h\iint_{\Omega_s}W^2dxdy-\frac{1}{2}m_0\omega^2(W(x_0,y_0)+Z)^2+\frac{1}{2}\bar{k}_0Z^2 \qquad (1)$$

Bunda $W=W(x,y)$ – plastinka o‘rta sirtining ko‘chish amplitudasi; Z – osilgan massaning ko‘chish amplitudasi; ν_2 – Puasson koeffitsiyentlaridan biri; $\bar{D}_1,\bar{D}_2$ – egilishdagi kompleks formadagi bikrliklar; $\bar{D}_\kappa$ – ortotrop plastinka uchun bosh yo‘nalishda buralishdagi kompleks bikrlik:

$$\bar{D}_1=\frac{\bar{E}_1h^3}{12(1-\nu_1\nu_2)},\bar{D}_2=\frac{\bar{E}_2h^3}{12(1-\nu_1\nu_2)},\bar{D}_k=\frac{\bar{G}_kh^3}{12},$$

$$\bar{E}_1=E_{01}[1-\Gamma_{1E}^c(\omega_R)-i\,\Gamma_{1E}^s(\omega_R)],$$
$$\bar{E}_2=E_{02}[1-\Gamma_{2E}^c(\omega_R)-i\,\Gamma_{2E}^s(\omega_R)],$$
$$\bar{G}_k=G_{0k}[1-\Gamma_{kG}^c(\omega_R)-i\,\Gamma_{kG}^s(\omega_R)], \qquad (2)$$
$$\Gamma_{E_1,E_2,G_k}^c(\omega)=\int_0^\infty R_{E_1,E_2,G_k}(\tau)\cdot cos\,\omega\,\tau d\tau\quad;$$
$$\Gamma_{E_1,E_2,G_k}^s(\omega)=\int_0^\infty R_{E_1,E_2,G_k}(\tau)\,sin\,\omega\,\tau d\tau.$$

Bu yerda: E_{01},E_{02} –oniy elastiklik moduli, G_{0k} – oniy siljish moduli, h – ortotrop plastinkaning qalinligi, $\bar{k}_0$ – deformatsiyalanuvchi prujinaning oniy bikrlik koeffitsiyenti, ν_1,ν_2 – ortotrop plastinka uchun Puasson koeffitsiyentlari, $R_{E_1,E_2,G_k}(\tau)$ – relaksatsiya yadrolari.

Izotrop materiallar uchun (izotrop plastinka) quyidagi munosabatlar bajariladi:

$$E_{01}=E_{02}=E_0,\nu_1=\nu_2=\nu,D_{01}=D_{02}=D_0,$$
$$\bar{D}_k=\frac{\bar{D}}{2}(1-\nu) \qquad (3)$$

Masalani yechishda o‘lchamsiz parametrlardan foydalanamiz,

$$\boldsymbol{x=\frac{\tilde{x}}{a},y=\frac{\tilde{y}}{b},x_0=\frac{\tilde{x}_0}{a},y_0=\frac{\tilde{y}_0}{b},}$$

buning uchun quyidagicha o‘lchamsiz parametr olamiz $\varepsilon = \frac{a}{b}$. Tenglamani yechish uchun plastinka salqiligi ifodasida quyidagicha o‘zgaruvchilarni ajratib olamiz:

$$W(x,y) \cong \sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn} X_m(x) Y_n(y). \tag{4}$$

Agar plastinkani tomonlari oddiygina tayanchlarga tayangan bo‘lsa, u holda

$$X_m(x) = \sin m\pi x,\ \ Y_n(y) = \sin n\pi y \tag{5}$$

Agar plastinkani tomonlari qistirilgan bo‘lsa, u holda quyidagi munosabatlar o‘rinli bo‘ladi

$$\begin{aligned} X_m(x) &= (\sin k_m x - shk_m x)\frac{\cos k_m - chk_m}{\sin k_m - shk_m}, \\ Y_n(y) &= (\sin k_n y - shk_n y)\frac{\cos k_n - chk_n}{\sin k_n - shk_n}. \end{aligned} \tag{6}$$

Bu yerda k_s – kompleks parametrli transsendent tenglamaning

$$\cos k\, chk - 1 = 0$$

ildizlari.

Qo‘yilgan massalani yechish uchun Reley-Rits usulini qo‘llab (1), funksionalni A_{mn} koeffitsiyent bo‘yicha minimizasiya qilamiz. Xuddi shunday massani siljish **Z** bo‘yicha ham qo‘yamiz:

$$\begin{aligned} &\frac{\partial J[W]}{\partial A_{ql}} = \frac{\bar{D}_1}{ab}\sum_{m=1}^{M_m}\sum_{m=1}^{M_m} A_{mn}\varepsilon^{-2}\int_{A_n}\frac{d^2X_m}{dx^2}Y_n\frac{d^2X_q}{dx^2}Y_l dxdy + \\ &+\frac{\bar{D}_1}{ab}\sum_{m=1}^{M_m}\sum_{m=1}^{M_m} A_{mn}\nu_2\int_{A_n}\left[\frac{d^2X_m}{dy^2}X_q\frac{d^2Y_l}{dy^2}Y_n + \frac{d^2X_q}{dx^2}X_m\frac{d^2Y_n}{dy^2}Y_l\right]dxdy + \\ &+\frac{\bar{D}_1}{ab}\sum_{m=1}^{M_m}\sum_{m=1}^{M_m} A_{mn}\varepsilon^2\frac{\bar{D}_2}{\bar{D}_1}\int_{A_n}\frac{d^2Y_n}{dy^2}X_m\frac{d^2Y_l}{dy^2}Y_q dxdy + \\ &+\frac{\bar{D}_1}{ab}\sum_{m=1}^{M_m}\sum_{m=1}^{M_m} A_{mn}\frac{4\bar{D}_k}{\bar{D}_1}\int_{A_n}\frac{dX_m}{dx}\frac{dY_n}{dy}\frac{dX_q}{dx}\frac{dY_l}{dy}dxdy - \\ &-\rho hab\omega^2\frac{m_0}{m_p}((\textstyle\sum_{m=1}^{M_m}\sum_{n=1}^{N_n} A_{mn}X_q(x_0)X_m(x_0)Y_l(y_0)Y_n(y_0)) + ZX_q(x_0)Y_l(y_0)) = 0, \end{aligned} \tag{8}$$

$$\frac{\partial J[Z]}{\partial Z} = Z\bar{k}_0 - \rho hab\omega^2\frac{m_0}{m_p}((\textstyle\sum_{m=1}^{M_m}\sum_{n=1}^{N_n} A_{mn}X_m(x_0)Y_n(y_0)) + Z) = 0. \tag{9}$$

Bu (4) kompleks parametrli algebraik bir jinsli tenglamalar sistemasi noldan farqli yechimga ega bo‘lishi uchun uning asosiy aniqlovchisi noldan farqli bo‘lishi kerak. Bu shartdan matritsa ko‘rinishidagi quyidagicha transsendent tenglamani olamiz:

$$|C^{\cdot}(\Omega_R) - \Omega^2 M^{\cdot}| = 0, \tag{10}$$

bunda

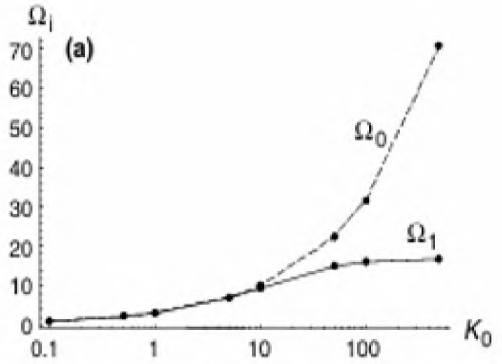
$$\begin{aligned} &\Omega_i^2 = \frac{\rho h}{\bar{D}_1}a^4\omega_i^2, C^{\cdot} = \begin{vmatrix} C & 0 \\ 0 & \varepsilon^{-1}\bar{K}_0 \end{vmatrix}, C = [c_{qlmn}] \\ &M^{\cdot} = \begin{bmatrix} M & \varepsilon^{-2}\frac{m_0}{m_p}X_q(x_0)Y_l(y_0) \\ \varepsilon^{-2}\frac{m_0}{m_p}X_m(x_0)Y_n(y_0) & \varepsilon^{-2}\frac{m_0}{m_p} \end{bmatrix}, M = [m_{qlmn}] \end{aligned} \tag{11}$$

$$\begin{aligned} c_{qlmn} &= \varepsilon^{-1}\int_{A_n}\left(\frac{d^2X_q}{dx^2}\frac{d^2X_m}{dx^2}\right)(Y_jY_m)dxdy + \\ &+\nu_2\int_{A_n}\left(\left(X_q\frac{d^2X_m}{dx^2}\right)\left(\frac{d^2Y_l}{dy^2}Y_n\right) + \left(X_m\frac{d^2X_q}{dx^2}\right)\left(\frac{d^2Y_n}{dy^2}Y_l\right)\right)dxdy + \\ &+\varepsilon^2\frac{\bar{D}_2}{\bar{D}_1}\int_{A_n}(X_qX_m)\left(\frac{d^2Y_l}{dy^2}\frac{d^2Y_n}{dy^2}\right)dxdy + \frac{4\bar{D}_k}{\bar{D}_1}\int_{A_n}\left(\frac{dX_q}{dx}\frac{dX_m}{dx}\left(\frac{dY_l}{dy}\frac{dY_n}{dy}\right)\right)dxdy, \end{aligned}$$

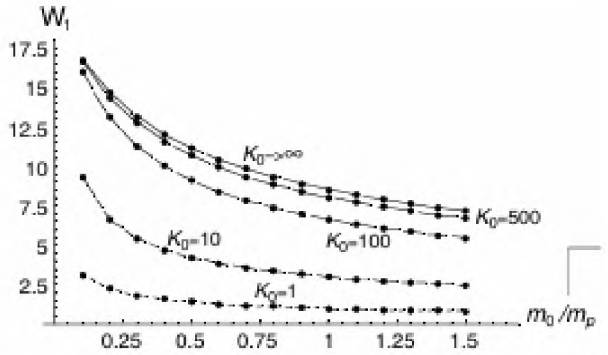
$$c_{qlmn} = \varepsilon^{-2} \int (X_q X_m)(Y_l Y_n) dxdy, \bar{K}_0 = \frac{ab}{D_{01}} \bar{k}_0, m_p = \rho hab$$

Qo‘yilgan masala [6,7]da keltirilgan metodikaga asosan yechiladi.

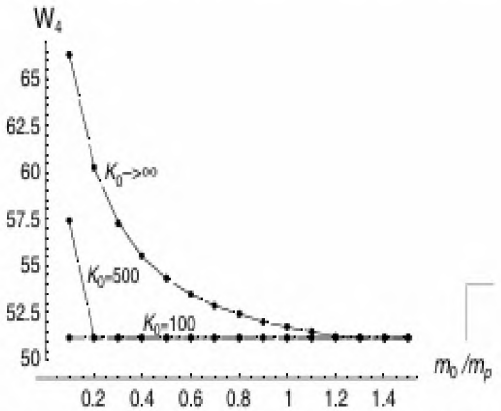
3. Sonli natijalar va ularning tahlili. Yuqoridagi 1-rasmda keltirilgan qovushoq- elastik plastinkaga osilgan massadan iborat mexanik sistemaning erkin tebranishlari massalasi ko‘raylik. Natijalar 2 – 5-rasmlarda keltirilgan. Plastinkaning atrofi qistirilgan. Plastinkaga m_0 massa qovushoq-elastik prujinka orqali osilgan. Massa osilgan joyining koordinatasi (x_0, y_0) bilan belgilangan. Massa plastinkani $x_0=y_0=2/3$ masofasida joylashgan. Hisoblashlarda kuchsiz singular uch parametrli relaksasiya yadrosi (Koltunov-Rjanisin yadrosi)dan foydalanildi: $R_k(t) = A_k e^{-\beta_k t} / t^{1-\alpha_k}$. Quyidagi parametrlar qabul qilingan: $A = 0{,}048;\ \beta = 0{,}05;\ \alpha = 0{,}1$. Plastinkaning materiali ortotrop qovushoq-elastik bo‘lsin. Osilgan massa $m = m_0 / m_p = 0.25$ bo‘lsin.



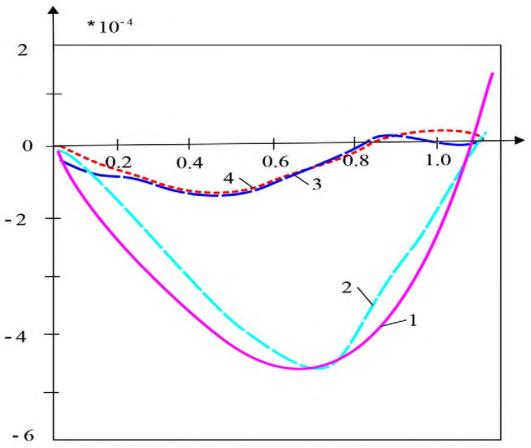
2-rasm. Chastota ikkita modasi haqiqiy qismining elastik elementning oniy elastiklik moduliga bog‘liq o‘zgarishi (m=0.12).



3-rasm. Birinchi tebranishlar formasiga ($x_0=y_0=1/2$) mos kelgan ko‘chishning massalar nisbatiga bog‘liq o‘zgarishi (m=0.12).



4-rasm. To‘rtinchi tebranishlar formasiga ($x_0=y_0=1/2$) mos kelgan ko‘chishning massalar nisbatiga bog‘liq o‘zgarishi (m=0.12).



5-rasm. Plastinkaga yopishtirilgan massaning tebranishlar formasi.

$1.\omega^{*} = 0.075; 2.\omega^{*} = 0.090; 3.\omega^{*} = 0.095; 4.\omega^{*} = 0.12; 5.\omega^{*} = 0.15.$

Massalar nisbatini o‘zgarishi chastotalar haqiqiy qismlarining o‘zgarishiga sabab bo‘lar ekan. Materiallarning qovushoqlik xossalarini hisobga olish ko‘chish va kuchlanishlarni 10–15%ga kamaytirishga imkon beradi.

Xulosalar.

1. Qovushoq elastik ortotrop plastinkaga osilgan plastinkani xos tebranishlari massalasini yechish metodikasi asosida chastota transendent tenglamasi olindi. Osilgan massa va oniy elastiklik modulini chastotalar va tebranishlar formasiga ta'siri parametrik o‘rganildi. Sonli natijalalar olinib, tahlil qilindi.

2. Olingan natijalardan ko‘rinib turibdiki, osiladigan massalar nisbatining o‘zgarishiga plastinka va massalarda bo‘ladigan ko‘chishlar amplitudasi kuchli bog‘liq bo‘lar ekan.

Foydalanilgan adabiyotlar ro‘yxati:

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О ПРОДОЛЖЕНИИ СУММЫ РЯДА ХАРТОГСА С ГАРМОНИЧЕСКИМИ КОЭФФИЦИЕНТАМИ

Аннотация. *В данной работе рассматривается вопрос о продолжении сумм рядов Гартогса, допускающих гармоническое продолжение вдоль фиксированного направления, предполагая только лишь гармоничность коэффициентов. Доказывается аналог основной леммы Гартогса для таких рядов.*

Ключевые слова: *гармоническая функция, сепаратно гармонические функции, особая точка, гармоническое продолжение, ряд Гартогса.*