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INVARIANTS IN SPECIAL LINEAR TRANSFORMATIONS ON A PLANE

Annotation. *In this article, quantities preserved under special linear transformations are studied. The geometry of a special linear transformation is explored, and it is proven that under this transformation, the product of the semi-axes of curves-such as ellipses, hyperbolas, and special hyperbolas- remains constant.*

Key words*: ellipse; hyperbola; special hyperbola; linear transformation; invariant.*

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TEKISLIKDA MAXSUS CHIZIQLI ALMASHTIRISHDAGI INVARIANTLAR

Annotasiya. *Ushbu maqolada maxsus chiziqli almashtirishda saqlanuvchi kattaliklar o'rganilgan. Maxsus chiziqli almashtirishning geometriyasi o'rganilib, bu almashtirishda ellipslar, giperbolar va maxsus giperbolar kabi egri chiziqlarning yarim o'qlarining ko'paytmasi o'zgarmasligi isbotlangan.*

Kalit so'zlar: *ellips; giperbola, maxsus giperbola, chiziq almashtirish, invariant.*

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ИНВАРИАНТЫ В СПЕЦИАЛЬНЫХ ЛИНЕЙНЫХ ПРЕОБРАЗОВАНИЯХ НА ПЛОСКОСТИ

Аннотация. *В статье изучаются величины, сохраняющиеся при особых линейных пре-образованиях. Исследуется геометрия специального линейного преобразования и доказано, что при этом преобразовании произведение полуосей кривых, таких как эллипсы, гиперболы и специальные гиперболы, остается постоянным.*

Ключевые слова*. эллипс; гипербола; специальная гипербола; линейная преобразование; инвариант.*

Introduction. The concept of invariant is one of the important concepts in mathematics. An invariant refers to a quantity or algebraic expression related to a line that remains unchanged when a transformation is applied to the line. We consider the operation (motion) involved in the transformation. Invariants are used to solve problems related to lines. When changing the parameter of a line, that is, replacing one parameter with another, invariant quantities remain preserved.

Preliminaries

Let's consider the following linear transformation in the Euclidean plane:

$$\begin{cases} x' = x + a, \\ y' = hx + y + b. \end{cases} \tag{1}$$

This transformation consists of a translation parallel to $\vec{a} = (a,b)$ vector and $A = \begin{pmatrix} 1 & 0 \\ h & 1 \end{pmatrix}$ matrix transformation [1,2]. Here $DetA = 1$. This A matrix is an element of the Heisenberg group [3,4].

In a (1) linear transformation, let a=b=0. Then:

$$\begin{cases} x' = x, \\ y' = hx + y. \end{cases} \qquad 0 \le h < \infty \tag{2}$$

If we take $x = x_0$ line parallel to the Oy axis, then our linear transformation is as follows:

$$\begin{cases} x' = x_0, \\ y' = hx_0 + y. \end{cases}$$

This implies that the $x' = x_0$ equality represents the Oy axis unchanged under the transformation and $y' = hx_0 + y$ equality represents that the Ox axis shifts by a distance of hx_0 along itself [5] (see Figure 1).

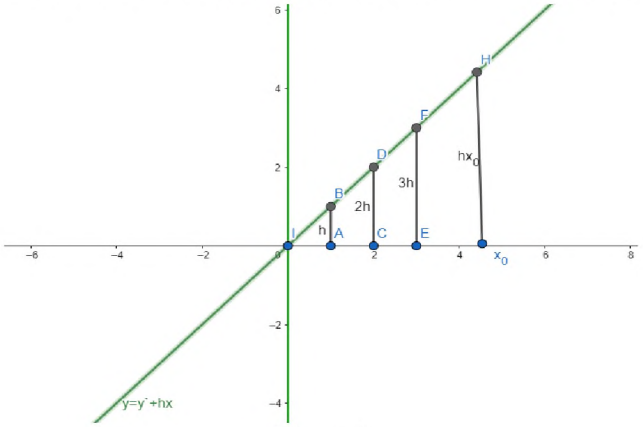


Figure1

Defination 1. Let the quadratic curve be given by the following equation [6]:

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0. \tag{3}$$

In that case, the following

$$(a_{11}l + a_{12}m)x + (a_{12}l + a_{22}m)y + a_{13}l + a_{22}m = 0$$

line is called conjugate diameter to {l,m} direction of quadratic curve for {l,m} non-specific direction.

Similarly, if the $\{l', m'\}$ and $\{l, m\}$ directions have conjugate diameters, the following equality holds:

$$l' : m' = -(a_{12}l + a_{22}m) : (a_{11}l + a_{12}m).$$

Defination 2. [7] Orthogonal and conjugate directions are referred to as the principal directions of a quadratic curves.

In this case, the condition for orthogonality is:

$$ll' + mm' = 0. \tag{4}$$

From condition (4), it follows that for the principal directions of a curve given by equation (3), the following equality holds [8]:

$$a_{12}l^2 + lm(a_{22} - a_{11}) - a_{12}m^2 = 0. \tag{5}$$

My results. Let an ellipse be given by the following formula:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \tag{6}$$

Let's consider how this formula changes in the

$$\begin{cases} x = x', \\ y = y' + hx', \end{cases}$$

transformation.

$$\frac{x'^2}{a^2} + \frac{(y' + hx')^2}{b^2} = 1$$

$$\frac{x'^2}{a^2}+\frac{y'^2+h^2x'^2+2hx'y'}{b^2}=1$$

$$\left(\frac{1}{a^2}+\frac{h^2}{b^2}\right)x'^2+\frac{2h}{b^2}x'y'+\frac{1}{b^2}y'^2-1=0$$

$$\left(b^2+a^2h^2\right)x'^2+2a^2hx'y'+a^2y'^2-a^2b^2=0 \qquad (7)$$

(7) equation represents a quadratic curve.

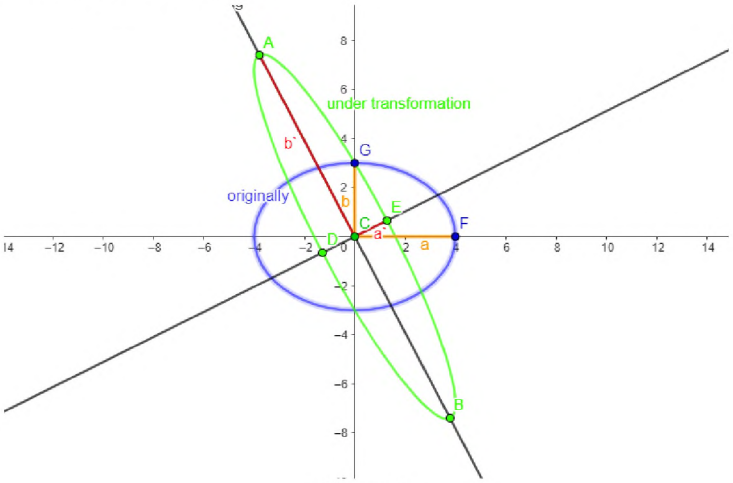


Figure 2.

The coefficients of this quadratic curve are given below:

$$a_{11}=b^2+a^2h^2,\quad a_{12}=a^2h,\quad a_{22}=a^2$$

Now, let's find the angle coefficients of the principal directions of the quadratic curve.
From the above, we find the principal directions. In this case, the following equality holds:

$$(a_{11}l+a_{12}m)(-m)+(a_{12}l+a_{22}m)l=0$$

$$-a_{12}m^2+(a_{22}-a_{11})ml+a_{12}l^2=0\,|:(-l^2)$$

$$a_{12}\left(\frac{m}{l}\right)^2+(a_{11}-a_{22})\frac{m}{l}-a_{12}=0$$

$$k_1=tg\varphi=\frac{m}{l}$$

$$a_{12}tg^2\varphi+(a_{11}-a_{22})tg\varphi-a_{12}=0$$

We substitute the coefficients from this formula and determine the angle coefficients.

$$a^2h*tg^2\varphi+\left(b^2+a^2h^2-a^2\right)*tg\varphi-a^2h=0$$

$$D=\left(b^2+a^2h^2-a^2\right)^2+4a^4h^2 \qquad (8)$$

$$tg\varphi=\frac{-\left(b^2+a^2h^2-a^2\right)+\sqrt{D}}{2a^2h}$$

Since the two principal directions are orthogonal, the condition

$$k_1*k_2=-1$$

holds for their angle coefficients k_1,k_2 . From this,

$$k_2 = -\frac{1}{k_1} \qquad k_2 = -\frac{1}{tg\varphi} = -ctg\varphi$$

$$ctg\varphi = \frac{1}{tg\varphi} = \frac{2a^2h}{\sqrt{D} - \left(b^2 + a^2h^2 - a^2\right)} = \frac{2a^2h\left(\sqrt{D} + \left(b^2 + a^2h^2 - a^2\right)\right)}{D - \left(b^2 + a^2h^2 - a^2\right)^2} = \frac{2a^2h\left(\sqrt{D} + \left(b^2 + a^2h^2 - a^2\right)\right)}{4a^4h^2}$$

$$ctg\varphi = \frac{\sqrt{D} + \left(b^2 + a^2h^2 - a^2\right)}{2a^2h}$$

Principal directions: $y = tg\varphi \cdot x$ and $y = -ctg\varphi \cdot x$

We find the x_1', x_2', which are intersection points of the ellipse and the principal lines by substituting them into equation (7). These intersection points' coordinates are: $A_1\left(x_1', y_1'\right), B_1\left(x_2', y_2'\right)$.

Let the semi-major and semi-minor axes of the ellipse generated by the operation be a′ and b′, respectively. A′ represents the distance from $A_1\left(x_1', y_1'\right)$ to O(0,0), and b` represents the distance from $B_1\left(x_2', y_2'\right)$ to O(0,0).

Accordingly, a′ and b′ are obtained as follows:

$$a' = \sqrt{x_1'^2 + y_1'^2} = \sqrt{x_1'^2 + tg^2\varphi * x_1'^2} = x_1'\sqrt{1 + tg^2\varphi} = \frac{x_1'}{\cos\varphi} \Rightarrow x_1' = a'\cos\varphi$$

$$b' = \sqrt{x_2'^2 + y_2'^2} = \sqrt{x_2'^2 + ctg^2\varphi * x_2'^2} = x_2'\sqrt{1 + ctg^2\varphi} = \frac{x_2'}{\sin\varphi} \Rightarrow x_2' = b'\sin\varphi \quad (9)$$

We find the intersection points principal lines and equation (7):

$$\begin{cases} \left(b^2 + a^2h^2\right)x_1^2 + 2a^2h * tg\varphi x_1'^2 + a^2tg^2\varphi x_1'^2 = a^2b^2 \\ \left(b^2 + a^2h^2\right)x_2^2 + 2a^2h * (-ctg\varphi)x_2'^2 + a^2ctg^2\varphi x_2'^2 = a^2b^2 \end{cases}$$

We substitute the values from formula (9) for x_1', x_2'

$$\begin{cases} \left(b^2 + a^2h^2 + 2a^2htg\varphi + \alpha^2tg^2\varphi\right)a'^2\cos^2\varphi = a^2b^2 \\ \left(b^2 + a^2h^2 - 2a^2hctg\varphi + \alpha^2ctg^2\varphi\right)b'^2\sin^2\varphi = a^2b^2 \end{cases}$$

We multiply the system and find the product a′b′.

$$\left(b^2 + a^2h^2 + 2a^2htg\varphi + \alpha^2tg^2\varphi\right)\left(b^2 + a^2h^2 - 2a^2htcg\varphi + \alpha^2ctg^2\varphi\right)\cos^2\varphi\sin^2\varphi\left(a'b'\right)^2 = a^4b^4 \qquad (10)$$

Currently,

$$\left(b^2 + a^2h^2 + 2a^2htg\varphi + \alpha^2tg^2\varphi\right)\left(b^2 + a^2h^2 - 2a^2htcg\varphi + \alpha^2ctg^2\varphi\right)\cos^2\varphi\sin^2\varphi$$

We simplify the expression

$$\left(b^2 + a^2h^2 + 2a^2htg\varphi + \alpha^2tg^2\varphi\right)\left(b^2 + a^2h^2 - 2a^2htcg\varphi + \alpha^2ctg^2\varphi\right) = b^4 + a^2h^2b^2 -$$

$$-2a^2hb^2ctg\varphi + a^2b^2ctg^2\varphi + \alpha^2h^2b^2 + a^4h^4 - 2a^4h^3ctg\varphi + \alpha^4h^2ctg^2\varphi + 2a^2b^2htg\varphi +$$

$$+2a^4h^3tg\varphi - 4a^4h^2 + 2a^4hctg\varphi + a^2b^2tg^2\varphi + a^4h^2tg^2\varphi - 2a^4htg\varphi + \alpha^4 =$$

$$= \left(a^4h^2 + a^2b^2\right)\left(tg^2\varphi + ctg^2\varphi\right) + (2a^4h^3 + 2a^2hb^2 - 2a^4h) + \left(tg\varphi - ctg\varphi\right) +$$

$$+\left(b^4 + a^4 + a^4h^4 + 2a^2h^2b^2 - 4a^4h^2\right) = \left(a^4h^2 + a^2b^2\right)\left(tg\varphi - ctg\varphi\right)^2 +$$

$$+2a^2h\left(a^2h^2 + b^2 - a^2\right)\left(tg\varphi - ctg\varphi\right) + \left(b^4 + a^4 + a^4h^4 + 2a^2h^2b^2 + 2a^2b^2 - 2a^4h^2\right)$$

$$tg\varphi - ctg\varphi = \frac{-2\left(b^2 + a^2h^2 - b^2\right)}{2a^2h} = -\frac{\left(b^2 + a^2h^2 - a^2\right)}{a^2h}$$

$$\cos^2\varphi\sin^2\varphi=\frac{tg^2\varphi}{\left(1+tg^2\varphi\right)^2}=\frac{1}{\left(tg\varphi+ctg\varphi\right)^2}=\frac{1}{\frac{D}{a^4h^2}}=\frac{a^4}{\left(b^2+a^2h^2-a^2\right)^2+4a^4h^2}$$

$$\left(b^2+a^2h^2+2a^2htg\varphi+\alpha^2tg^2\varphi\right)\left(b^2+a^2h^2-2a^2htcg\varphi+\alpha^2ctg^2\varphi\right)\cos^2\varphi\sin^2\varphi=$$

$$=\left[\left(a^4h^2+a^2b^2\right)\frac{\left(b^2+a^2h^2-a^2\right)^2}{a^4h^2}-2a^2h\left(a^2h^2+b^2-a^2\right)\frac{\left(a^2h^2+b^2-a^2\right)}{a^2h}+\left(a^2h^2+b^2-a^2\right)^2+4a^2b^2\right]*$$

$$*\frac{a^4h^2}{\left(a^2h^2+b^2-a^2\right)^2+4a^4h^2}=\left[\left(a^2h^2+b^2-a^2\right)^2\left(\frac{b^2}{a^2h^2}\right)+4a^2b^2\right]*\frac{a^4h^2}{\left(a^2h^2+b^2-a^2\right)^2+4a^4h^2}=$$

$$=\frac{b^2\left(\left(a^2h^2+b^2-a^2\right)^2+4a^4h^2\right)}{a^2h^2}*\frac{a^4h^2}{\left(a^2h^2+b^2-a^2\right)^2+4a^4h^2}=a^2b^2$$

So,

$$\left(b^2+a^2h^2+2a^2htg\varphi+\alpha^2tg^2\varphi\right)\left(b^2+a^2h^2-2a^2htcg\varphi+\alpha^2ctg^2\varphi\right)\cos^2\varphi\sin^2\varphi=a^2b^2$$

We substitute this into equation (10)

$$a^2b^2*\left(a'b'\right)^2=a^4b^4\Rightarrow\left(a'b'\right)^2=\frac{a^4b^4}{a^2b^2}=a^2b^2\Rightarrow\left(a'b'\right)^2=a^2b^2$$

$$\Rightarrow a'b'=ab.$$

Therefore, in the above transformation, $a`b`=ab$ remains unchanged. Namely, it is invariant.

It is known from the course of analytic geometry that the area of an ellipse is calculated using the formula $S=\pi ab$, where a and b are the semi-major and semi-minor axes, respectively.

Since the product ab remains unchanged in the above transformation, the area of the ellipse also remains constant. Namely, $S_1=S_2$. Here S_1 is area of originally ellipse, S_2 is area of under transformation ellipse (See Figure 3).

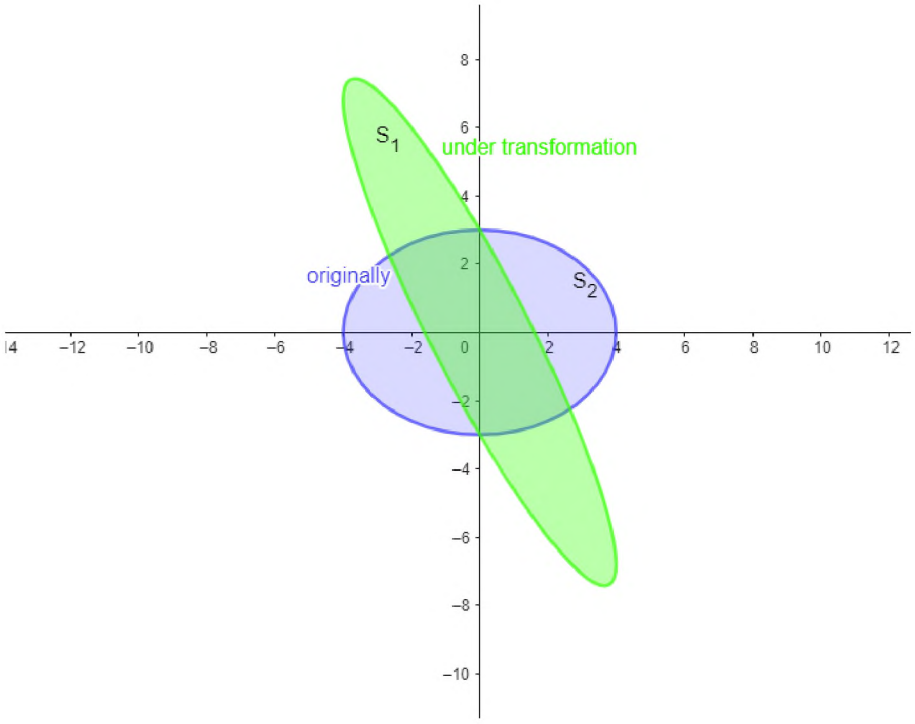


Figure 3.

Let a hyperbola be given by the following equation:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1. \tag{11}$$

We will observe the changes of this hyperbola under the

$$\begin{cases} x = x', \\ y = hx' + y', \end{cases} \tag{12}$$

transformation.

To do this, we substitute equation (12) into equation (11).

$$\begin{aligned} &\frac{x'^2}{a^2}-\frac{(hx'+y')^2}{b^2}=1 \\ &\frac{x'^2}{a^2}-\frac{\left(h^2x'^2+2hx'y'+y'^2\right)}{b^2}=1 \\ &\left(b^2-a^2h^2\right)x'^2-2a^2hx'y'-a^2y'^2-a^2b^2=0 \end{aligned} \tag{13}$$

Equation (13) represents a quadratic curve.

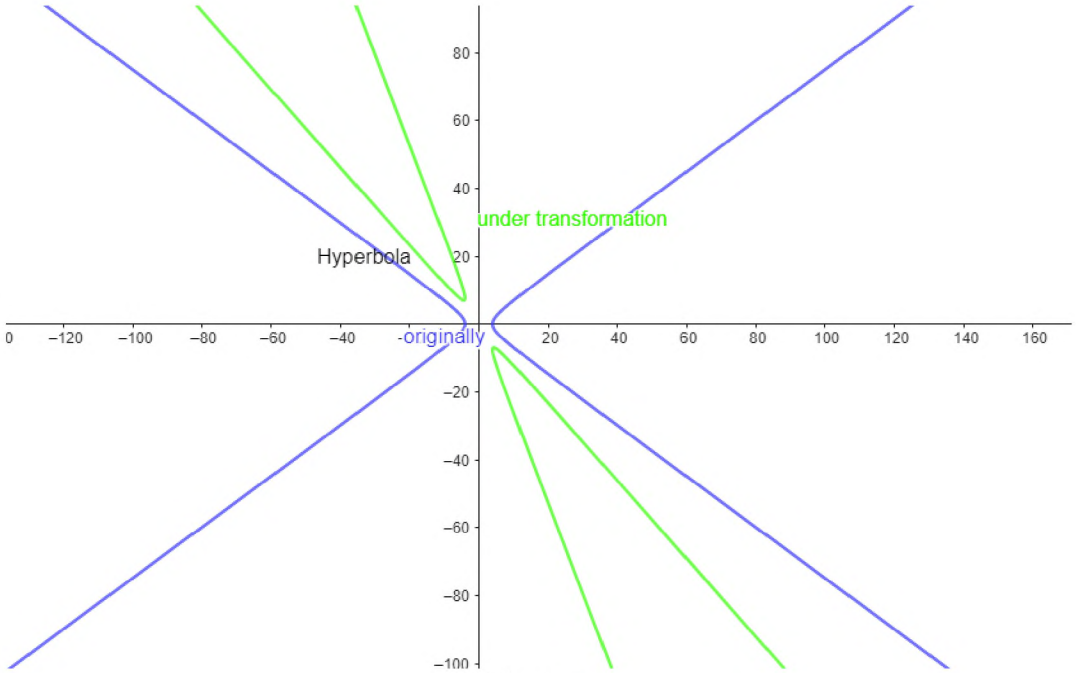


Figure 4.

The coefficients of this quadratic curve are as follows:

$$a_{11} = b^2 - a^2h^2$$
$$a_{12} = -a^2h$$
$$a_{22} = -a^2$$

We rotate this quadratic curve according to the principal axes. Now, we find the angle coefficients of the principal axes. Accordingly:

$$a_{12}tg^2\varphi + (a_{11} - a_{22})tg\varphi - a_{12} = 0.$$

We substitute the coefficients from this formula:

$$-a^2h*tg^2\varphi+\left(b^2-a^2h^2+a^2\right)*tg\varphi+a^2h=0$$

$$a^2htg^2\varphi-\left(b^2-a^2h^2+a^2\right)tg\varphi-a^2h=0$$

$$D=\left(b^2-a^2h^2+a^2\right)^2+4a^4h^2$$

$$tg\varphi=\frac{\left(b^2-a^2h^2+a^2\right)+\sqrt{D}}{2a^2h}$$

$$ctg\varphi=\frac{-\left(b^2-a^2h^2+a^2\right)+\sqrt{D}}{2a^2h}$$

Now we perform the rotation according to the principal axes as follows:

$$x'=x''\cos\varphi-y''\sin\varphi$$
$$y'=x''\sin\varphi+y''\cos\varphi. \qquad (14).$$

We substitute equation (14) into equation (13).

$$\left(b^2-a^2h^2\right)\left(x''\cos\varphi-y''\sin\varphi\right)^2-2a^2h\left(x''\cos\varphi-y''\sin\varphi\right)\left(x''\sin\varphi+y''\cos\varphi\right)-$$

$$-a^2\left(x''\sin\varphi+y''\cos\varphi\right)^2=a^2b^2$$

$$\left(\left(b^2-a^2h^2\right)\cos^2\varphi-2a^2h\sin\varphi\cos\varphi-a^2\sin^2\varphi\right)x''^2+(\ -2\left(b^2-a^2h^2\right)\cos\varphi\sin\varphi-2a^2h\cos^2\varphi+$$

$$+2a^2h\sin^2\varphi-2a^2\sin\varphi\cos\varphi\ )x''y''+\left(\left(b^2-a^2h^2\right)\sin^2\varphi+2a^2h\sin\varphi\cos\varphi-a^2\cos^2\varphi\right)y''^2-a^2b^2=0$$

In this quadratic curve, the coefficients of x″y″ become zero:

$$-2\left(b^2-a^2h^2\right)\cos\varphi\sin\varphi-2a^2h\cos^2\varphi+2a^2h\sin^2\varphi-2a^2\sin\varphi\cos\varphi=\left(a^2h^2-b^2\right)\sin2\varphi-$$

$$-2a^2h\cos2\varphi-a^2\sin2\varphi=\left(a^2h^2-a^2-b^2\right)\sin2\varphi-2a^2h\cos2\varphi=\Rightarrow$$

$$\sin2\varphi=\frac{2tg\varphi}{1+tg^2\varphi}=\frac{2}{tg\varphi+\mathrm{c}tg\varphi}=\frac{2a^2h}{\sqrt{D}}$$

$$\cos2\varphi=\frac{1-tg^2\varphi}{1+tg^2\varphi}=\frac{\mathrm{c}tg\varphi-tg\varphi}{\mathrm{c}tg\varphi+tg\varphi}=\frac{-\left(b^2+a^2-a^2h^2\right)}{\sqrt{D}}$$

$$\Rightarrow=-\left(a^2+b^2-a^2h^2\right)\frac{2a^2h}{\sqrt{D}}+2a^2h\frac{\left(b^2+a^2-a^2h^2\right)}{\sqrt{D}}=0$$

Therefore, the equation takes the following canonic form:

$$\left(\left(b^2-a^2h^2\right)\cos^2\varphi-2a^2h\sin\varphi\cos\varphi-a^2\sin^2\varphi\right)x''^2+\left(\left(b^2-a^2h^2\right)\sin^2\varphi+2a^2h\sin\varphi\cos\varphi-a^2\cos^2\varphi\right)y''^2=a^2b^2$$

we divide all to a^2b^2:

$$\frac{\left(\left(b^2-a^2h^2\right)\cos^2\varphi-2a^2h\sin\varphi\cos\varphi-a^2\sin^2\varphi\right)}{a^2b^2}x''^2+\frac{\left(\left(b^2-a^2h^2\right)\sin^2\varphi+2a^2h\sin\varphi\cos\varphi-a^2\cos^2\varphi\right)}{a^2b^2}y''^2=1$$

This is a hyperbola, and if we denote the real semi-axis as a` and the imaginary semi-axis as b`, then the following equations hold:

$$\frac{1}{a'^2}=\frac{\left(b^2-a^2h^2\right)\cos^2\varphi-\alpha^2h\sin2\varphi-a^2\sin^2\varphi}{a^2b^2}$$

$$\frac{1}{b'^2}=-\frac{\left(b^2-a^2h^2\right)\sin^2\varphi+\alpha^2h\sin2\varphi-a^2\cos^2\varphi}{a^2b^2}$$

We multiply the two equations.

$$\frac{1}{(a'b')^2}=-\frac{\left(\left(b^2-a^2h^2\right)\cos^2\varphi-\alpha^2h\sin 2\varphi-a^2\sin^2\varphi\right)\left(\left(b^2-a^2h^2\right)\sin^2\varphi+\alpha^2h\sin 2\varphi-a^2\cos^2\varphi\right)}{a^4b^4} \quad (15)$$

Then we simplify the numerator of this expression:

$$-\left(\left(b^2-a^2h^2\right)\cos^2\varphi-\alpha^2h\sin 2\varphi-a^2\sin^2\varphi\right)\left(\left(b^2-a^2h^2\right)\sin^2\varphi+\alpha^2h\sin 2\varphi-a^2\cos^2\varphi\right)=$$

$$=\left(\left(b^2+a^2-a^2h^2\right)\cos^2\varphi-a^2h\sin 2\varphi-a^2\right)\left(\left(b^2+a^2-a^2h^2\right)\sin^2\varphi+\alpha^2h\sin 2\varphi-a^2\right)=$$

$$=-\left(\frac{b^2-a^2-a^2h^2}{2}+\frac{b^2+a^2-a^2h^2}{2}*\frac{-\left(b^2+a^2-a^2h^2\right)}{\sqrt{D}}-a^2h\frac{2a^2h}{\sqrt{D}}\right)*$$

$$\left(\frac{b^2-a^2-a^2h^2}{2}-\frac{b^2+a^2-a^2h^2}{2}\frac{-\left(b^2+a^2-a^2h^2\right)}{\sqrt{D}}+a^2h\frac{2a^2h}{\sqrt{D}}\right)=$$

$$=-\left(\frac{b^2-a^2-a^2h^2}{2}+\frac{-\left(b^2+a^2-a^2h^2\right)^2-4a^4h^2}{2\sqrt{D}}\right)\left(\frac{b^2-a^2-a^2h^2}{2}+\frac{\left(b^2+a^2-a^2h^2\right)^2+4a^4h^2}{2\sqrt{D}}\right)=$$

$$=-\left(\frac{b^2-a^2-a^2h^2}{2}\right)^2+\frac{\left(\left(b^2+a^2-a^2h^2\right)^2+4a^4h^2\right)^2}{4D}=-\frac{\left(b^2-a^2-a^2h^2\right)^2}{4}+\frac{\left(b^2+a^2-a^2h^2\right)^2+4a^4h^2}{4}=$$

$$=\frac{-\left(b^2-a^2-a^2h^2\right)^2+\left(b^2+a^2-a^2h^2\right)^2+4a^4h^2}{4}=\frac{2a^2\left(2b^2-2a^2h^2\right)+4a^4h^2}{4}=\frac{4a^2b^2+4a^4h^2-4a^4h^2}{4}=$$

$$=a^2b^2$$

Therefore, the numerator of (15) is equal to a^2b^2 :

$$\frac{1}{(a'b')^2}=\frac{a^2b^2}{a^4b^4}\Rightarrow\frac{1}{(a'b')^2}=\frac{1}{a^2b^2}\Rightarrow a^2b^2=a'^2b'^2\Rightarrow a`b`=ab$$

Therefore, in the above operation, the product $a`b`=ab$ is preserved. Namely, it is invariant.

Let us be given a hyperbola with $y=\frac{k}{x}$ equation. This hyperbola is defined at $x\neq 0$

From this:

$$yx=k\ \rightarrow\ yx-k=0 \quad (16)$$

(16) equation represents a quadratic curve,

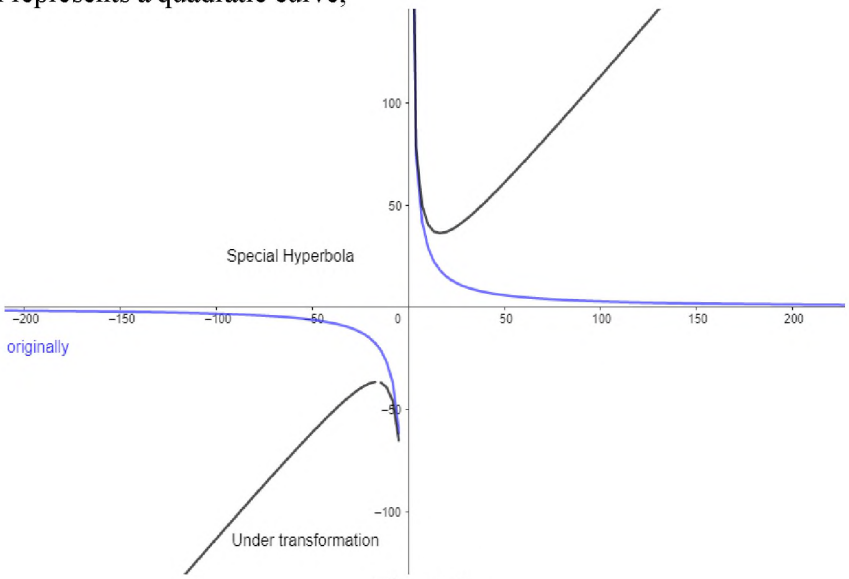


Figure 5.

and its coefficients are as follows:

$$a_{11} = 0$$
$$a_{12} = \frac{1}{2}$$
$$a_{22} = 0$$

We will rotate this hyperbola along the principal directions:

$$a_{12}tg^2\varphi + (a_{11} - a_{22})tg\varphi - a_{12} = 0 \Rightarrow$$
$$\Rightarrow \frac{1}{2}tg^2\varphi - \frac{1}{2} = 0 \Rightarrow tg^2\varphi = 1.$$

If k>0, we take $tg\varphi = 1$; if k<0, we take $tg\varphi = -1$.In this case, we assume k>0.

$$tg\varphi = 1 \Rightarrow \sin\varphi = \cos\varphi = \frac{1}{\sqrt{2}}$$

In (16)

$$\begin{cases} x = \frac{1}{\sqrt{2}}(x' - y') \\ y = \frac{1}{\sqrt{2}}(x' + y') \end{cases}$$

if we perform the substitution,

$$\frac{1}{2}(x' - y')(x' + y') = k \Rightarrow \frac{1}{2}x'^2 - \frac{1}{2}y'^2 = k$$

In the complete equation of the hyperbola, its semi-axes are

$$a = \sqrt{2k},$$
$$b = \sqrt{2k}.$$

In this case, the

$$a * b = 2k \tag{17}$$

equality holds.

Now we will observe how the hyperbola changes under the

$$\begin{cases} x = x', \\ y = hx' + y', \end{cases} \tag{18}$$

transformation. To do this, we will substitute (18) into (16).

It will take the form of $x'(hx' + y') = k \Rightarrow hx'^2 + x'y' - k = 0$

(19)

The coefficients are:

$$a_{11}' = h$$
$$a_{12}' = \frac{1}{2} \tag{20}$$
$$a_{22}' = 0$$

Let`s rotate this quadratic curve along the its principal direction. Here`s the formula for finding the principal direction:

$$a_{12}tg^2\varphi + (a_{11} - a_{22})tg\varphi - a_{12} = 0$$

We substitute the coefficients, which are given by (20) identity into the formula above:

$$\frac{1}{2}tg^2\varphi + htg\varphi - \frac{1}{2} = 0 \Rightarrow tg\varphi = \sqrt{h^2 + 1} - h$$

We perform

$$\begin{cases} x' = x''\cos\varphi - y''\sin\varphi \\ y' = x''\sin\varphi + y''\cos\varphi \end{cases} \quad (21)$$

transformation and rotate along the principal direction. In this process, we substitute equation (21) into equation (19):

$$h(x''\cos\varphi - y''\sin\varphi)^2 + (x''\cos\varphi - y''\sin\varphi)(x''\sin\varphi + y''\cos\varphi) - k = 0$$

$$(h\cos^2\varphi + \cos\varphi\sin\varphi)x''^2 + (\cos^2\varphi - \sin^2\varphi - 2h\cos\varphi\sin\varphi)x''y'' + (h\sin^2\varphi - \sin\varphi\cos\varphi)y''^2 - k = 0$$

In this equation, the coefficients of **x``y``** become zero:

$$(\cos^2\varphi - \sin^2\varphi - 2h\cos\varphi\sin\varphi) = \cos 2\varphi - h\sin 2\varphi = \frac{1-tg^2\varphi}{1+tg^2\varphi} - \frac{2htg\varphi}{1+tg^2\varphi} = \frac{2h\sqrt{h^2+1} - 2h^2 - 2h\sqrt{h^2+1} + 2h^2}{1+tg^2\varphi} = 0$$

Therefore, the quadratic curve`s equation takes the following canonic form:

$$(h\cos^2\varphi + \cos\varphi\sin\varphi)x''^2 - (\sin\varphi\cos\varphi - h\sin^2\varphi)y''^2 - k = 0$$

We multiply to 2:

$$(h(1+\cos 2\varphi) + \sin 2\varphi)x''^2 - (\sin 2\varphi - h(1-\cos 2\varphi))y''^2 - 2k = 0$$

We define semi-axes of hyperbola, which are generated under transformation as a` and b`. The following equality holds:

$$\begin{cases} \left(\frac{1}{a'}\right)^2 = \frac{h(1+\cos 2\varphi) + \sin 2\varphi}{2k} \\ \left(\frac{1}{b'}\right)^2 = \frac{\sin 2\varphi + h\cos 2\varphi - h}{2k} \end{cases}$$

We will multiply the two equalities:

$$\left(\frac{1}{a'b'}\right)^2 = \frac{1}{4k^2}\left((h\cos 2\varphi + \sin 2\varphi)^2 - h^2\right) \quad (22)$$

equality is obtained.

Now, we will simplify

$$(h\cos 2\varphi + \sin 2\varphi)^2 - h^2 \quad (23)$$

$$\cos 2\varphi = \frac{1-tg^2\varphi}{1+tg^2\varphi} = \frac{1-\left(\sqrt{h^2+1}-h\right)^2}{2\sqrt{h^2+1}\left(\sqrt{h^2+1}-h\right)} = \frac{2h\sqrt{h^2+1}-2h^2}{2\sqrt{h^2+1}\left(\sqrt{h^2+1}-h\right)} = \frac{h}{\sqrt{h^2+1}}$$

$$\sin 2\varphi = \frac{2tg\varphi}{1+tg^2\varphi} = \frac{2\left(\sqrt{h^2+1}-h\right)}{2\sqrt{h^2+1}\left(\sqrt{h^2+1}-h\right)} = \frac{1}{\sqrt{h^2+1}}$$

We will substitute these equalities into expression (23).

$$(h\cos 2\varphi + \sin 2\varphi)^2 - h^2 = \left(h*\frac{h}{\sqrt{h^2+1}} + \frac{1}{\sqrt{h^2+1}}\right)^2 - h^2 = \left(\frac{h^2+1}{\sqrt{h^2+1}}\right)^2 - h^2 = \left(\sqrt{h^2+1}\right)^2 - h^2 = h^2+1-h^2 = 1$$

Therefore, $(h\cos 2\varphi + \sin 2\varphi)^2 - h^2 = 1$

For this, the expression (22) will take the following form:

$$\left(\frac{1}{a'b'}\right)^2 = \frac{1}{4k^2} \Rightarrow (a'b')^2 = 4k^2 \Rightarrow a'b' = 2k$$

$$a'b' = 2k$$

In equation (17), was $ab = 2k$. So,

$$ab = a'b'$$

In the above transformation, the product ab does not change. Namely, it is invariant.

Similarly, it is shown for $k < 0$.

Conclusion. The obtained results show that when a quadratic curve experiences the transformation (2), causing the product of their semi-axes remain constant. In transformation (2) of quadratic curves, this provides the deformation of the shape in Euclidean space. Therefore, in this deformation, the product of the semi-axes remains unchanged.

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EFFECTS OF A MULTICOMPONENT HEAT SOURCE ON AMBIENT DENSITY IN MULTIDIMENSIONAL FIELDS

Annotation. *This paper investigates the effect of heat dissipation on medium density in multicomponent nonlinear heat dissipation processes in multidimensional fields. In order to solve the equation of linear heat distribution, a density and heat source of the built environment, taking into account the characteristics of reaction-diffusion processes are observed. obtained results: an estimation for the double linear heat diffusion equation, a model by homogeneity was constructed. The program code is compiled in the Python programming language and the process is modeled.*

Keywords: *Heat temperature, system, ambient speed, heat source, the density of the environment, power of the heat source, parabolic equation.*

KO'P KOMPONENTLI CHIZIQLI BO'LMAGAN ISSIQLIK TARQALISH JARAYONLARIDA ISSIQLIK TARQALISHINING O'RTA ZICHLIKKA TA'SIRI

Annotatsiya. *Ushbu maqola ko'p o'lchovli sohalarda ko'p komponentli chiziqli bo'lmagan issiqlik tarqalish jarayonlarida issiqlik tarqalishining o'rta zichlikka ta'sirini o'rganadi. Chiziqli issiqlik taqsimoti tenglamasini hal qilish uchun reaksiya-diffuziya jarayonlarining xususiyatlarini hisobga olgan holda qurilgan muhitning zichligi va issiqlik manbayi kuzatiladi. Olingan natijalar: qo'sh chiziqli issiqlik diffuziya tenglamasini baholash, bir jinslilik bo'yicha model tuzildi. Dastur kodi Python dasturlash tilida tuzilgan va jarayon modellashtirilgan.*

Kalit so'zlar: *Issiqlik harorati, sistema, atrof-muhit tezligi, issiqlik manbayi, atrof-muhitning zichligi, issiqlik manbasining kuchi, parabolik tenglama.*

ВЛИЯНИЕ ТЕПЛООТВОДА НА ПЛОТНОСТЬ СРЕДЫ В МНОГОКОМПОНЕНТНЫХ НЕЛИНЕЙНЫХ ПРОЦЕССАХ

Аннотация. *В данной работе исследуется влияние теплоотвода на плотность среды в многокомпонентных нелинейных процессах теплоотвода в многомерных областях. Для решения уравнения линейного распределения тепла наблюдают плотность застроенной среды и источника тепла с учетом свойств реакционно-диффузионных процессов. Полученные результаты: оценено билинейное уравнение теплодиффузии, построена модель однородности. Программный код компилируется на языке программирования Python и моделируется процесс.*