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YONISH JARAYONLARINI TAVSIFLOVCHI NODIVERGENT KO‘RINISHDAGI NOCHIZIQLI PARABOLIK TENGLAMALAR SISTEMASINING CHEGARALANMAGAN YECHIMLARINI BAHOLASH

Annotatsiya. *Ushbu maqolada yutilish va zichlikka ega nodivergent ko‘rinishdagi nochiziqli parabolik tenglamalar sistemasi uchun Koshi masalasi qaraladi, unga mos avtomodel tenglamalar keltirib chiqarilgan hamda yuqori yechim uchun baholar olingan.*

Kalit so‘zlar: *matematik modellashtirish, avtomodel tahlil, parabolik tenglamalar sistemasi, nodivergent shakl, yuqori yechimlar.*

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ESTIMATES OF UNBOUNDED SOLUTIONS OF A NON-DIVERGENT NONLINEAR PARABOLIC SYSTEM DESCRIBING THE PROCESSES OF COMBUSTION

Annotation. *In this paper, the Cauchy problem for a nonlinear parabolic system in non-divergence form with absorption and density is considered, the corresponding self-similar equations are derived, and estimates for a supersolution are obtained.*

Keywords: *mathematical modeling, self-similar analysis, system of parabolic equations, non-divergence form, supersolution.*

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ОЦЕНКИ НЕОГРАНИЧЕННЫХ РЕШЕНИЙ СИСТЕМЫ НЕЛИНЕЙНЫХ ПАРАБОЛИЧЕСКИХ УРАВНЕНИЙ В НЕДИВЕРГЕНТНОЙ ФОРМЕ, ОПИСЫВАЮЩИХ ПРОЦЕССЫ ГОРЕНИЯ

Аннотация. *В данной работе рассматривается задача Коши для нелинейной параболической системы в недивергентной форме с поглощением и плотностью; выводятся соответствующие автомодельные уравнения, а также получены оценки для верхнего решения.*

Ключевые слова: *математическое моделирование, автомодельный анализ, система параболических уравнений, недивергентная форма, верхнее решение.*

1. Kirish Ushbu ishda $Q = \{(t,x) : t > 0, x \in R\}$ sohada nodivergent ko‘rinishdagi nochiziqli parabolik tenglamalar sistemasi uchun quyidagi Koshi masalasi qaraladi:

$$\begin{cases} |x|^{l} \dfrac{\partial u}{\partial t} = u^{\alpha_1} \dfrac{\partial}{\partial x}\left(u^{m-1} \dfrac{\partial u}{\partial x} \right) - |x|^{l} v^{\beta_1}, \\ |x|^{l} \dfrac{\partial v}{\partial t} = v^{\alpha_2} \dfrac{\partial}{\partial x}\left(v^{m-1} \dfrac{\partial v}{\partial x} \right) - |x|^{l} u^{\beta_2}, \end{cases} \tag{1}$$

$$\begin{cases} u(0,x) = u_0(x) \geq 0 \\ v(0,x) = v_0(x) \geq 0 \end{cases} \tag{2}$$

bu yerda $\beta_1 > 1,\ \beta_2 > 1,\ m > 1,\ \alpha_1 > 1,\ \alpha_2 > 1,\ l > 0$ - sonli parametrlar, $u = u(t,x) \geq 0, v = v(t,x) \geq 0$ esa izlanayotgan yechimlar.

Ushbu maqolada yutilish va zichlikka ega bo‘lgan, nodivergent ko‘rinishdagi nochiziqli parabolik tenglamalar sistemasi o‘rganiladi. Sistemaning avtomodel yechimlarini topiladi va avtomodel tahlil asosida yechimlarning yuqori yechimlarini quriladi. Bunday sistemalar nochiziqli diffuziya muhim rol o‘ynaydigan issiqlik o‘tkazuvchanlik, yonish va kimyoviy reaksiyalar modellarida tabiiy ravishda yuzaga keladi. Qaralayotgan sistema yonish jarayonidagi harorat o‘zgarishi va reaktiv modda konsentratsiyasini tavsiflab, nochiziqli diffuziya va issiqlik ajralib chiqishining asosiy xususiyatlarini o‘zida aks ettiradi.

[1] ishda muallif $t \to \infty$ da $\Omega \times (0,\infty)$ sohada $u_t = \Delta u^m$ tenglama uchun $t > 0$, $i = 1,K, i_0$ larda $u(a_i,t) = \infty$, Ω da $u(x,0) = u_0(x)$ va $\partial\Omega \times (0,\infty)$ da $u = f$ shartlarni qanoatlantiruvchi,

chekli sondagi nuqtalarda blow-up u yechimning (yechimlarni chekli vaqtda cheksizga intilishini) asimptotik xatti-harakatini isbotlagan. Shuningdek, muallif chegaralangan silindrik sohada mos yon chegaraviy qiymatlarga ega bo'lgan va $t \to \infty$ da ikkita berilgan garmonik funksiya orasida tebranadigan chekli sondagi buzilish nuqtalariga ega bo'lgan yechimni qurgan hamda $\Omega \times (0,\infty)$ da $u_t = \Delta u^m$, Ω da $u(x,0) = u_0(x)$, $\forall t > 0, i = 1,2,\ldots,i_0$ larda $u(a_i,t) = \infty$ va $\partial\Omega \times (0,\infty)$ da $u = \infty$ bo'lgan minimal yechimning mavjudligini isbotlagan.

[2] maqolada mualliflar o'zaro bog'langan nochiziqli parabolik sistemalar sinfi uchun Koshi masalasini o'rganib, ychimlarning asimptotik xatti-harakatini tahlil qilganlar. Integral baholashlar va mos yuqori yechimlar yordamida Fujita tipidagi blow-up teoremasi isbotlangan. Bundan tashqari, diffuziya va fazoviy o'lcham orqali aniqlanadigan kritik Fujita ko'rsatkichi topilgan.

[3] ishda manba hadi nostandart o'sish shartlarida anizotrop nochiziqlilikka ega parabolik masalalar o'rganilgan. O'zgaruvchi ko'rsatkichli Sobolev fazolarida kuchsiz yechimlarning mavjudligi va yagonaligi isbotlangan. Blow-up (yechimlarni chekli vaqtda cheksizga intilishini) hodisalarini tahlil qilish uchun energiya usullari qo'llanilgan bo'lib, ular ychimlarning ham manfiy, ham musbat boshlang'ich energiya holatlarida blow-up bo'lishi mumkinligini ko'rsatadi. O'zgaruvchi ko'rsatkichlar va koeffitsiyentlarning Fujita tipidagi buzilish jarayonida muhim rol o'ynashi ko'rsatilgan.

[4]-ishda manbada kross-diffuziyaga ega, nodivergent ko'rinishdagi nochiziqli parabolik sistemaning blow-up yechimlarining ba'zi xossalari tekshiriladi. Mos yordamchi funksiyalarni qurish va analitik usullarni qo'llash orqali buziluvchi yechimlarning mavjudligi va xatti-harakati uchun yetarli shartlar keltirib chiqariladi. Olingan natijalar nochiziqli diffuziya dinamikasini chuqurroq tushunishga hissa qo'shadi hamda ko'p o'lchovli va murakkabroq sistemalarni keyingi tadqiq qilish uchun asos bo'lib xizmat qiladi.

[5]ga ko'ra maqolada manba hadi va nochiziqli chegaraviy oqimga ega bo'lgan, nodivergent ko'rinishdagi ikki karra nochiziqli parabolik tenglamaning kritik eksponentalari o'rganilgan. Global yechim mavjudligining kritik eksponentasi ham, kritik Fujita eksponentasi ham keltirib chiqarilgan. Maqolada ko'rsatilgan tenglamalar sistemasi o'ziga xos xususiyati shundaki, u $u = 0$ va $\nabla u = 0$ nuqtalarda buziladi (degeneratsiyaga uchraydi). Natijada klassik yechimlar mavjud bo'lmasligi mumkin, shuning uchun faqat tavsiflangan umumlashgan ma'nodagi kuchsiz yechimlar qaralgan.

Boshqa sistemalar [6, 11] avvalroq nochiziqli parabolik tenglamalar hamda divergent ko'rinishdagi tenglamalar sistemalari uchun ma'lum bo'lgan nochiziqli ajratish (splitting) usuli, asimptotik nazariya va turli almashtirishlarga asoslangan asimptotik usullar orqali o'rganilgan. Nodivergent ko'rinishdagi nochiziqli parabolik tenglamalar sistemasi uchun avtomodel yechimlarning asimptotikasi qurilgan.

[8,9] manbalarda sekin va tez diffuziya holatlarini hisobga olgan holda nodivergent ko'rinishdagi degenerativ parabolik sistema uchun avtomodel yechimlarning asimptotik xatti-harakati tekshirilgan. Taqqoslash usulidan foydalanib, nodivergent ko'rinishdagi kross-diffuziyali parabolik sistema uchun Koshi masalasida buzilish tarqalishining chekli tezligi (FSPD) xossasi isbotlangan. Olingan yechimlarning asimptotik xatti-harakatiga asoslanib, sonli parametrlarning qiymatlariga qarab, sekin va tez diffuziya rejimlari bo'yicha iteratsion jarayon uchun mos boshlang'ich yaqinlashishlar taklif qilingan.

Shuningdek, ba'zi kitob va ilmiy maqolalar sohada qo'llaniladigan tenglamalar va usullarni hozirgi kunda mavjud bo'lgan boshqa adabiyotlarga qaraganda ancha kengroq qamrab oladi. Ushbu qo'llanmaga chiziqli va nochiziqli tenglamalarni yechish hamda tahlil qilish uchun ishlatiladigan aniq, asimptotik, taqribiy analitik va sonli usullari kiritilgan. Mualliflar, shuningdek, issiqlik o'tkazuvchanligi, elastiklik, gidrodinamika va boshqa turli xil amaliy masalalarda yuzaga keladigan ko'plab tenglamalar hamda ularning yechimlarini samarali qurish uchun teoremalarni taqdim etganlar [10;15-b.].

Ushbu maqolada, avtomodel yondashuvdan foydalanib, (1)-sistemaning xususiy yechimi olingan. Ushbu maqolaning asosiy maqsadi avtomodel tahlil [1;15-b.] asosida (1)–(2) masala uchun yuqori ychim mavjud bo'lishining yetarli shartlarini keltirib chiqarishdan iborat.

2. Masalaning avtomodel tahlili. Quyida biz tenglamalar sistemasi avtomodel ko'rinishini keltirib chiqarish uchun nochiziqli ajratish usulidan [1,6,7,12] foydalanamiz. (1)-sistemaning avtomodel yechimlarini qurish uchun $u(t,x)$ va $v(t,x)$ funksiyalarini quyidagi ko'rinishda qaraymiz:

$$\begin{cases} u(t,x) = \bar{u}(t) w(x,\tau(t)), \\ v(t,x) = \bar{v}(t) \varphi(x,\tau(t)). \end{cases} \tag{3}$$

Faraz qilaylik, $\mathrm{u}(\mathrm{t})$ va $v(\mathrm{t})$ quyidagi ko‘rinishga ega bo‘lsin:

$$\overline{u}(t)=A_1(T-t)^{\frac{\beta_1+1}{1-\beta_1\beta_2}},\quad \overline{v}(t)=A_2(T-t)^{\frac{\beta_2+1}{1-\beta_1\beta_2}},$$

bu yerda $A_i=\left(\frac{\beta_i+1}{\beta_1\beta_2-1}\right)^{\frac{1}{\beta_1\beta_2-1}}\left(\frac{\beta_{3-i}+1}{\beta_1\beta_2-1}\right)^{\frac{\beta_i}{\beta_1\beta_2-1}}, i=1,2.$

(3)-ifodadan foydalanib, (1)-sistemani quyidagi ko‘rinishga keltirish mumkin:

$$\begin{cases} |x|^{l}\dfrac{\partial w}{\partial\tau}=w^{\alpha_1}\dfrac{\partial}{\partial x}\left(w^{m-1}\dfrac{\partial w}{\partial x}\right)-d_1\tau^{-1}(\psi^{\beta_1}+w)|x|^{l}, \\ |x|^{l}\dfrac{\partial \psi}{\partial\tau}=\psi^{\alpha_2}\dfrac{\partial}{\partial x}\left(\psi^{m-1}\dfrac{\partial \psi}{\partial x}\right)-d_2\tau^{-1}(w^{\beta_2}+\psi)|x|^{l}. \end{cases} \tag{4}$$

bu yerda

$$\tau(t)=\begin{cases} -A_1^{\alpha_1+m-1}\dfrac{(T-t)^{n_1(\alpha_1+m-1)+1}}{n_1(\alpha_1+m-1)+1}, & \text{agar} \quad n_1(\alpha_1+m-1)+1<0, \\ -A_1^{\alpha_1+m-1}\ln(T-t), & \text{agar} \quad n_1(\alpha_1+m-1)+1=0. \end{cases}$$

$$n_2(\alpha_2+m-1)=n_1(\alpha_1+m-1),\qquad \kappa=A_2^{\alpha_2+m-1}A_1^{1-\alpha_1-m},$$

$$d_1=\frac{n_1}{n_1(\alpha_1+m-1)+1},\ d_2=\frac{n_2\kappa}{n_2(\alpha_2+m-1)+1}.$$

Biz masalani $n_1(\alpha_1+m-1)+1<0$ holat uchun qaraymiz. (4)-sistemaning avtomodel yechimini quyidagi ko‘rinishda qidiramiz:

$$w(\tau,x)=f(\xi),\quad \psi(\tau,x)=\phi(\xi),\quad \xi=\frac{x}{\tau^{l+2}} \tag{5}$$

(5) ni (4)-sistemaga qo‘yib, quyidagi avtomodel tenglamalarni hosil qilamiz:

$$\begin{cases} f^{\alpha_1}\dfrac{d}{d\xi}\left(f^{m-1}\dfrac{df}{d\xi}\right)+\dfrac{1}{l+2}\xi^{l+1}\dfrac{df}{d\xi}-d_1\xi^{l}\left(\phi^{\beta_1}+f\right)=0 \\ \phi^{\alpha_2}\dfrac{d}{d\xi}\left(\phi^{m-1}\dfrac{d\phi}{d\xi}\right)+\dfrac{1}{l+2}\xi^{l+1}\dfrac{d\phi}{d\xi}-d_2\xi^{l}\left(f^{\beta_2}+\phi\right)=0 \end{cases} \tag{6}$$

3. Masalaning yuqori yechimi. Faraz qilaylik, (6)-tenglamalar sistemasi uchun quyidagi shartlar bajarilsin: $f'(0)=0,\phi'(0)=0$.

U holda, $\overline{f}(\xi)$ va $\overline{\phi}(\xi)$ funksiyalar quyidagi ko‘rinishga ega bo‘lsin:

$$\overline{f}(\xi)=B_1(a-\xi^{l+2})^{c_1},\ \overline{\phi}(\xi)=B_2(a-\xi^{l+2})^{c_2} \tag{7}$$

bu yerda $a>0, c_i=\dfrac{1}{\alpha_i+m-1}, B_i=\left[\dfrac{c_i-d_i}{c_i(l+2)(c_im(l+2)-1)}\right]^{c_i}$ (i=1,2) funksiyalar (6)-tenglamalar sistemasini qanoatlantiradi.

U holda, yuqori yechim haqidagi quyidagi teorema o‘rinli.

1-teorema. Faraz qilaylik, quyidagilar o‘rinli bo‘lsin:

$$1.\ n_i(\alpha_i+m-1)+1<0,$$

$$2.\ B_i^{\alpha_i+m-1}<\frac{1}{(l+2)^2(c_im-1)},\quad (i=1,2),$$

$$3.\ u_+(0,x)\ge u_0(x),\ v_+(0,x)\ge v_0(x),\quad x\in\mathbb{R}.$$

U holda, (1)-(2) masala uchun Q sohada global yechim mavjud va quyidagi baho o‘rinli: $u_+(t,x)\ge u(t,x), v_+(t,x)\ge v(t,x),\quad x\in\mathbb{R}$, bu yerda

$$\begin{cases} u_+(t,x)=\overline{u}(t)\overline{f}(\xi), \\ v_+(t,x)=\overline{v}(t)\overline{\phi}(\xi). \end{cases} \tag{8}$$

Isbot. Teoremani isbotlash uchun yechimlarni taqqoslash usulidan foydalanamiz. Shunday qilib, taqqoslash usuliga, asosan, mos $u(t)$, $v(t)$ funksiyalari olinadi. (8) ni (1) ga qo‘yib, quyidagi tengsizliklarni hosil qilish mumkin:

$$\begin{cases} \overline{f}^{\alpha_1}\dfrac{d}{d\xi}\left(\overline{f}^{m-1}\dfrac{df}{d\xi}\right)+\dfrac{1}{l+2}\xi^{l+1}\dfrac{d\overline{f}}{d\xi}-d_1\xi^l\left(\overline{\phi}^{\beta_1}+\overline{f}\right)\le 0 \\ \overline{\phi}^{\alpha_2}\dfrac{d}{d\xi}\left(\overline{\phi}^{m-1}\dfrac{d\phi}{d\xi}\right)+\dfrac{1}{l+2}\xi^{l+1}\dfrac{d\overline{\phi}}{d\xi}-d_2\xi^l\left(\overline{f}^{\beta_2}+\overline{\phi}\right)\le 0 \end{cases} \tag{9}$$

Agar $\overline{f}(\xi)$ va $\overline{\phi}(\xi)$ funksiyalar (7) ko‘rinishda qaralsa, (9)-tengsizliklar sistemasini quyidagicha qayta yozish mumkin:

$$\begin{cases} aB_1^{\alpha_1+m}c_1(l+2)^2(c_1m-1)-aB_1c_1-d_1B_2^{\beta_1}(a-\xi^{l+2})^{l\beta_1-(c_1-1)}\le 0, \\ aB_2^{\alpha_1+m}c_2(l+2)^2(c_2m-1)-aB_2c_2-d_2B_1^{\beta_2}(a-\xi^{l+2})^{l\beta_2-(c_2-1)}\le 0. \end{cases} \tag{10}$$

Ma’lumki, $\overline{f}(\xi)\ va\ \overline{\phi}(\xi)$ musbat funksiyalar hamda $d_1,d_2>0$, shuning uchun

$$B_i^{\alpha_i+m-1}<\frac{1}{(l+2)^2(c_im-1)},\ i=1,2.$$

U holda, (1)-(2) masalaning yechimlarini taqqoslash haqidagi teoremaga asosan, Q sohada global yechim mavjud va quyidagi baho bajariladi:

$$u_+(t,x)\ge u(t,x),\quad v_+(t,x)\ge v(t,x)$$

Teorema isboti tugallandi.

Xulosa. Yonish jarayonlarini tavsiflovchi, yutilishga ega bo‘lgan nodivergent ko‘rinishdagi nochiziqli parabolik sistema uchun Koshi masalasi taqqoslash usuli yordamida o‘rganildi. Qaralayotgan masalaga mos yuqori baholar olindi. Ushbu natijalar nochiziqli diffuziya va issiqlik ajralishining yonish dinamikasi hamda harorat evolyutsiyasiga sezilarli ta’sir ko‘rsatishini namoyon etadi. Olingan natijalar issiqlik o‘tkazuvchanlik va yonish jarayonlarini modellashtiruvchi nochiziqli parabolik sistemalarni nazariy jihatdan rivojlantirishga o‘z hissasini qo‘shadi.

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О ПОДПРОСТРАНСТВАХ ПРОСТРАНСТВА ВЕРОЯТНОСТНЫХ МЕР, ЯВЛЯЮЩИХСЯ МНОГООБРАЗИЯМИ РАЗЛИЧНОЙ РАЗМЕРНОСТИ

Аннотация. *Исследуются геометрические и топологические свойства различных гомотопических плотных подмножеств, являющихся многообразиями различной размерности компакта $P(X)$. В работе доказано, что подпространства вида $P_n(X)\backslash P_{n-1}(X)$ (при $n > 1$) являются евклидовыми многообразиями размерности $n-1$. С помощью построения специальных гомотопий установлена гомотопическая плотность исследуемых подмножеств и проанализированы их геометрические свойства как Z-множеств в смысле Шори и Андерсона. Также показано, что для любого бесконечного компакта пространство вероятностных мер $P(X)$ удовлетворяет аксиоме отделимости S_4. Полученные результаты вносят вклад в теорию ковариантных функторов, подтверждая высокую степень топологической однородности структуры $P(X)$ и её естественных подпространств. В частности, обосновано, что пространство мер с конечными носителями $P_\omega(X)$ является абсолютным ретрактом (AR).*

Ключевые слова: *вероятностные меры, гомотопически плотно, многообразия, Z −множество.*