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MOSLANGAN MANBALI MANFIY TARTIBLI MODIFITSIRLANGAN KORTEVEG-DE FRIZ TENGLAMASINING BIR SOLITONLI YECHIMI

Annotatsiya. *Mazkur maqolada moslangan manbali manfiy tartibli modifitsirlangan Korteveg-de Friz (KdF) tenglamasining bir solitonli yechimi "tez kamayuvchi" funksiyalar sinfida sochilish nazarisining teskari masalasi usuli yordamida qurilgan.*

Kalit so‘zlar: *potensial, xos qiymat, xos funksiya, tez kamayuvchi funksiya, operator.*

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ОДНОСОЛИТОННОЕ РЕШЕНИЕ МОДИФИЦИРОВАННОГО УРАВНЕНИЯ КОРТЕВЕГА–ДЕ ФРИЗА ОТРИЦАТЕЛЬНОГО ПОРЯДКА С САМОСОГЛАСОВАННЫМ ИСТОЧНИКОМ

Аннотация. *В данной статье методом обратной задачи рассеяния построено односолитонное решение модифицированного уравнения Кортевега–де Фриза (мКдФ) отрицательного порядка с самосогласованным источником в классе быстро убывающих функций.*

Ключевые слова: *потенциал, собственное значение, собственная функция, быстроубывающая функция, оператор.*

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ONE-SOLITON SOLUTION OF THE NEGATIVE-ORDER MODIFIED KORTEWEG–DE VRIES EQUATION WITH A SELF-CONSISTENT SOURCE

Annotation. *In this paper, a one-soliton solution of the negative-order modified Korteweg–de Vries (mKdV) equation with a self-consistent source is constructed in the class of rapidly decreasing functions using the inverse scattering transform method.*

Key-words: *potential, eigenvalue, eigenfunction, rapidly decreasing function, operator.*

Kirish. Manfiy tartibli integrallanuvchi sistemalar sayoz suv to‘lqinlarining sinishi va cho‘qqisimon tuzilishga ega bo‘lgan shakllarini modellashtirishda muhim o‘rin tutadi. Shuningdek, ular nosilliq to‘lqin shakllarini tavsiflash imkonini berib, nochiziqli akustika hamda bosim to‘lqinlari dinamikasidagi jarayonlarni o‘rganishda samarali qo‘llaniladi. Xususan, manfiy tartibli modifitsirlangan Korteveg–de Friz (mKdF) tenglamasi ichki to‘lqinlar dinamikasi, nochiziqli panjara tebranishlari va magnitogidrodinamik jarayonlarni modellashtirishda, ayniqsa, energiyaning lokal to‘planishi, nochiziqli fokuslanish yoki anomal dispersiya ustunlik qiladigan rejimlarni tadqiq etishda yuzaga keladi. Mazkur tenglama, odatda, nolokal xarakterga ega bo‘lib, uning xossalari musbat tartibli analoglaridan sifat jihatidan sezilarli darajada farq qiladi. Jumladan, bunday tenglamalar nochiziqli dispersiya va to‘lqinlarning sinishi kabi hodisalarni namoyon qilishi mumkin. Shunga qaramay, manfiy tartibli mKdF tenglamasi o‘zining integrallanuvchanlik xususiyatini saqlab qoladi hamda Laks juftligi ko‘rinishidagi tasvirga ega bo‘ladi.

Nochiziqli evolyutsion tenglamaning integrallanuvchanligi solitonsimon yechimlarning mavjudligi hamda ularga sochilish nazariyasining teskari masalasi usulini qo‘llash mumkinligi bilan tavsiflanadi. Ushbu usul ayrim nochiziqli xususiy hosilali differensial tenglamalarni ularga mos chiziqli spektral sochilish nazariyasi masalasiga keltirish orqali aniq integrallash imkonini beradi.

Adabiyotlar tahlili. Sochilish nazariyasining teskari masalasi usuli dastlab K.S.Gardner, J.M. Greene, M.D.Kruskal va R.M.Miura [1], V.E.Zaxarov va A.B.Shabat [2], M.J.Ablowitz, D.J.Kaup, A.C. Nyuel va H.Segur [3]larning fundamental ishlarida matematik fizikaning nochiziqli klassik modellariga nisbatan ishlab chiqilgan. [4–5] ishlarda esa teskari sochilish usulining muhim fizik jarayon va hodisalarni matematik modellashtirishda uchraydigan keng sinfdagi klassik nochiziqli evolutsion tenglamalarga tatbiq etishdagi samaradorligi va universalligi yanada yaqqol namoyon etilgan.

J.M.Veroskiy [6], P.J.Olver kiritgan rekursiya operatorlarining manfiy darajalarini kiritdi hamda birinchilardan bo‘lib manfiy tartibli nochiziqli evolyutsion tenglama tushunchasini asosladi. A.M.Vazvaz esa [7]. J.M.Veroskiy g‘oyasidan foydalanib, manfiy tartibli KdF va mKdF tenglamalarini keltirib chiqardi hamda rekursiya operatorlari va ularga bog‘liq analitik usullar yordamida ularning integrallanuvchanligi va soliton yechimlarini tadqiq etdi. Nisbatan so‘nggi [8–9] ishlarda ham manfiy tartibli mKdF tenglamalariga sochilish nazariyasining teskari masalasi usulini qo‘llash, jumladan, ko‘p solitonli yechimlarni qurish hamda ularning Laks tasvirlarini tahlil qilish masalalari o‘rganilgan.

Tadqiqot metodologiyasi. Manfiy tartibli mKdV tenglamasining tez kamayuvchi yechimlarini qurish uchun Zaxarov–Shabat spektral masalasiga mos teskari sochilish usuli qo‘llaniladi. Teskari masala Gelfand–Levitan–Marchenko yondashuvi asosida yechiladi.

Tahlil va tadqiqot natijalari. Ushbu ishda quyidagi

$$\begin{cases} \eta_x - 2uu_t = -2\sum\limits_{k=1}^{2N} u(f_{k1}g_{k1} - f_{k2}g_{k2}), \\ u_{xt} + u + 2\eta u = \sum\limits_{k=1}^{2N} \dfrac{d}{dx}\left(f_{k1}g_{k1} - f_{k2}g_{k2}\right) \end{cases} \tag{1}$$

moslangan manbali manfiy tartibli mKdF tenglamasini

$$u(x,0) = u_0(x), \quad x \in R, \tag{2}$$

boshlang‘ich shart bilan qaraymiz. Bu yerda $u_0(x)$ haqiqiy funksiya bo‘lib,

$$\int_{\mathbb{R}} \left(1+|x|\right)^5 |u_0(x)|dx < \infty$$

shartni qanoatlantiradi.

Aytaylik, $L(0) = i\begin{pmatrix} \dfrac{d}{dx} & -u_0 \\ -u_0 & -\dfrac{d}{dx} \end{pmatrix}$ operator spektral maxsusliklarga ega emas hamda $2N$ ta $\xi_1(0), \xi_2(0), \ldots, \xi_{2N}(0)$ oddiy xos qiymatlarga ega bo‘lib, $\mathrm{Im}(\xi_k(0)) > 0$, $\xi_{N+k}(0) = -\xi_k(0)$, $k = 1,2,\ldots,N$, bo‘lsin.

Faraz qilaylik, $u(x,t)$ va $\eta(x,t)$ funksiyalar haqiqiy qiymatli silliq funksiyalar bo‘lib, quyidagi

$$\mathcal{L} = \left\{ \begin{array}{c} u(x,t) \in C^2(\mathbb{R}\times(0,\infty)) \cap C(\mathbb{R}\times[0,\infty)), \quad \eta(x,t) \in C^{1,0}(\mathbb{R}\times[0,\infty)) \\ \lim\limits_{x\to\pm\infty} \eta(x,t) = 0, \quad t \in \mathbb{R}; \\ \int\limits_{-\infty}^{+\infty} \left(1+|x|\right)\left(|u(x,t)| + |u_x(x,t)| + |u_t(x,t)| + |u_{xt}(x,t)|\right)dx < \infty, \quad t \in \mathbb{R} \end{array} \right\}$$

sinfga tegishli bo‘lsin. Shuningdek, (1) sistemada $f_k = \left(f_{k1}(x,t), f_{k2}(x,t)\right)^T$ funksiya $L(t)$ operatorning ξ_k xos qiymatiga mos xos funksiyasi, $g_k = \left(g_{k1}(x,t), g_{k2}(x,t)\right)^T$ esa $L(t)g_k = \xi_k g_k$ tenglamaning f_k bilan chiziqli erkli bo‘lgan yechimi bo‘lib,

$$W\{f_k, g_k\} \equiv f_{k1}g_{k2} - f_{k2}g_{k1} = \omega_k(t), \quad k = 1,2,\ldots,2N, \tag{3}$$

$$\lim_{\delta\to\infty} \int_{-\delta}^{\delta} \left(f_{k1}g_{k2} + f_{k2}g_{k1}\right)dx = A_k(t), \quad k = 1,2,\ldots,2N \tag{4}$$

shartlar bajarilsin. Bunda $A_k(t)$ va $\omega_k(t)$ funksiyalar noldan farqli haqiqiy uzluksiz funksiyalar bo‘lib, ushbu

$$\omega_k(t) = -\omega_{N+k}(t),\ A_k(t) = -A_{N+k}(t), \qquad \xi_n = -\xi_k,\ \ k = 1,2,\ldots,N$$

munosabatlar o‘rinli.

Qaralayotgan (1)-(4) Koshi masalasining bir solitonli yechimini $L(t)$ operator uchun teskari masala usulidan foydalanib qurishda quyidagi teorema natijalaridan foydalanamiz

1-teorema. Agar $\{u(x,t), \eta(x,t), f_k(x,t), g_k(x,t), k = 1,2,\ldots,N\}$ funksiyalar (1)-(4) masalaning $\mathcal{L}$ sinfdagi yechimi bo‘lsa, u holda $L(t)$ operatorning sochilish nazariyasi berilganlari t vaqt bo‘yicha quyidagicha o‘zgaradi:

$$\frac{\partial r^+(\xi,t)}{\partial t} = \left(\frac{i}{2\xi} + i\sum_{k=1}^{N}\omega_k(t)\left(\frac{1}{\xi-\xi_k} + \frac{1}{\xi_k+\xi}\right)\right) r^+(\xi,t),$$

$$\frac{d\xi_n(t)}{dt} = i\omega_n(t),\ \ \frac{db_n(t)}{dt} = \left(\frac{i}{2\xi_n} + i\beta_n(t)\omega_n(t)\right) b_n(t),\ \ n = 1,2,\ldots,N\ .$$

Moslangan manbali manfiy tartibli mKdF tenglamasiga qo‘yilgan (1)-(4) Koshi masalasini ushbu

$$u(x,0) = \frac{-2}{\cosh(2x)}$$

boshlang‘ich shart bilan qaraymiz. $t = 0$ da $L(0)$ operatori uchun to‘g‘ri masalani yechib, ushbu

$$r(x,0) = 0,\ \xi_1(0) = i,\ c_1(0) = 2i$$

natijalarni olamiz. U holda 1-teorema natijalaridan foydalanib, qaralayotgan masala uchun ushbu

$$u(x,t) = \frac{-2\mu}{\cosh(2\mu x - \sigma)},$$

potensialni hosil qilamiz, bu yerda $\xi_n = i\mu(t)$, $\mu(t) = 1 + \frac{1}{2}\int_0^t \omega_n(\tau)d\tau$, $\frac{d\sigma(t)}{dt} = \frac{1}{2\mu} + i\beta_n\omega_n$ bo‘lib, μ, β_n, ω_n, σ, $n = 1,2,\ldots,N$, - t ga bog‘liq haqiqiy funksiyalar. $L(t)$ operatorning ξ_n xos qiymatiga mos $f_k = f_k(x,\xi,t)$ xos funksiyasi va $\psi_k = \psi_k(x,\xi,t)$ Yost yechimi orasidagi $f_k = c_k^+\psi_k = c_k^-\phi_k$ proporsionallik munosabati hamda Yost yechimlarning integral tasviridan foydalanib,

$$f_{11} = \frac{c^+e^{-\mu x}}{2\cosh(2\mu x - \sigma)},\ f_{12} = \frac{c^+e^{\mu x - \sigma}}{2\cosh(2\mu x - \sigma)}$$

bo‘lishini topamiz. Aytaylik, $\chi_n(x) = \frac{d}{d\xi}(\phi - b_n\psi)\Big|_{\xi=\xi_n}$, $n = 1,\ldots,N$, funksiya $L\chi_n(x) = \xi_n\chi_n(x)$ tenglamaning yechimi bo‘lsin. U holda ushbu

$$\chi_n = \beta_n(t)\phi_n(x,t) + \gamma_n(t)g_n(x,t),\ \ n = 1,2,\ldots,N \qquad (5)$$

munosabat o‘rinli bo‘ladi, bu yerda $\gamma_n(t)$ va $\beta_n(t)$ funksiyalar x ga bog‘liq emas. (5) formulaga asosan,

$$g_{11} = \frac{\left((2i\mu\beta_1 - 4\mu x - 1)e^{-\mu x+\sigma} - e^{3\mu x-\sigma}\right)\omega_1}{2c^+\cosh(2\mu x - \sigma)},$$

$$g_{12} = \frac{\left((2i\mu\beta_1 - 4\mu x + 1)e^{\mu x} + e^{-3\mu x+2\sigma}\right)\omega_1}{2c^+\cosh(2\mu x - \sigma)}$$

natijalarni olamiz. (4) shartdan foydalanib, $\beta_1(t)$ ni topamiz. Olingan natijalar hamda (1) sistemaning 1-tenglamasiga asosan,

$$\eta(x,t)=\frac{-1}{\cosh^2(2\mu x-\sigma)}$$

funksiyani aniqlaymiz.

Xulosa. Ushbu ishda moslangan manbali manfiy tartibli mKdF tenglamasi uchun Koshi masalasi sochilish nazariyasining teskari masalasi usuli yordamida tadqiq etildi. Spektral berilganlarning vaqt bo‘yicha evolyutsiyasidan foydalanib, qaralayotgan tenglamaning bir solitonli yechimi oshkor ko‘rinishda qurildi. Olingan natijalar manfiy tartibli integrallanuvchi nochiziqli tenglamalar uchun sochilish nazariyasining teskari masalasi usulining samarali ekanligini ko‘rsatadi.

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SONNING BUTUN VA KASR QISMI FUNKSIYALARI ISHTIROK ETGAN AYRIM NOSTANDART TENGLAMALARNI YECHISHNING KLASSIK USULLARI

Annotatsiya. *Ushbu maqolada sonning butun va kasr qismi funksiyalari ishtirok etgan nostandart tenglamalarni yechishning klassik analitik hamda baholash usullari tadqiq qilingan. Ermit teoremasi va maxsus lemmalar yordamida murakkab algebraik tenglamalarni soddalashtirish va uning ildizlarini topish algoritmlari ko‘rsatib berilgan. Taqdim etilgan nazariy natijalar aniq amaliy masalalar yechimi bilan asoslangan.*

Kalit so‘zlar: *sonning butun qismi, kasr qismi, antye funksiyasi, Hermite teoremasi, nostandart tenglamalar, baholash usuli.*

КЛАССИЧЕСКИЕ МЕТОДЫ РЕШЕНИЯ НЕКОТОРЫХ НЕСТАНДАРТНЫХ УРАВНЕНИЙ, СОДЕРЖАЩИХ ФУНКЦИИ ЦЕЛОЙ И ДРОБНОЙ ЧАСТИ ЧИСЛА

Аннотация. *В данной статье исследованы классические аналитические методы и оценки для решения нестандартных уравнений, содержащих функции целой и дробной части числа. На основе теоремы Эрмита и специальных лемм продемонстрированы алгоритмы упрощения и нахождения корней сложных алгебраических уравнений. Теоретические результаты подтверждаются решениями конкретных практических задач.*

Ключевые слова: *целая часть числа, дробная часть числа, функция антье, теорема Эрмита, нестандартные уравнения, метод оценок.*