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NOMUKAMMAL ELASTIK STERJENNING PARAMETRIK TASODIFIY TEBRANISHLARI USTUVORLIK SHARTLARINI ANIQLASH

Annotatsiya. *Ushbu ishda nomukammal elastik sterjenning tasodifiy parametrik tebranishlari ustuvorligini tekshirish masalasi qaralgan. Ustuvorlik shartlari sistema parametrlari va o‘zgaruvchilariga bog‘liq holda analitik ko‘rinishda aniqlangan. Ito usulidan foydalanib, xarakteristik tenglama aniqlanib, ustuvorlik shartlari olingan.*

Kalit so‘zlar: *nomukammal elastik, sterjen, gisterezis, parametrik tasodifiy tebranish, o‘rtalashtirish, ustuvorlik sharti.*

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ОПРЕДЕЛЕНИЕ УСЛОВИЙ УСТОЙЧИВОСТИ ПАРАМЕТРИЧЕСКИХ СЛУЧАЙНЫХ КОЛЕБАНИЙ НЕСОВЕРШЕННО УПРУГОГО СТЕРЖНЯ

Аннотация. *В этой работе рассмотрена задача исследования устойчивости параметрических случайных колебаний несовершенно упругого стержня. Условие устойчивости определено в аналитическом виде в зависимости от параметров и переменных системы. С помощью метода Ито определено характеристическое уравнение и получены условия устойчивости.*

Ключевые слова*: несовершенно упругий стержень, гистерезис, параметрическое случайное колебание, усреднение, условие устойчивости.*

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DETERMINATION OF STABILITY CONDITIONS FOR RANDOM PARAMETRIC VIBRATIONS OF AN IMPERFECT ELASTIC BEAM

Annotation. *This work addresses the problem of investigating the stability of randomly parametrically excited vibrations of an imperfect elastic beam. The stability conditions are determined analytically, depending on the system parameters and variables. Using Ito‘s method, the characteristic equation is derived, and the stability conditions are obtained.*

Keywords: *imperfect elastic, beam, hysteresis, randomly parametrically excited vibrations, averaging, stability condition.*

Kirish. Mexanik sistemalar elementlari materiallaridagi chiziqlimas dissipativ xarakteristikalarini hisobga olgan holda, ularning tebranishlari va harakatlari ustuvorligini tekshirish muhim hisoblanadi.

[1–2] ishlarda garmonik va tasodifiy qo‘zg‘alishlar ta’sirlaridagi tebranishlardan himoyalanuvchi tekis taqsimlangan parametrli sistemalarning gisterezis tipidagi elastik dissipativ xarakteristikalarini hisobga olgan holdagi ko‘ndalang tebranishlari matematik modellashtirilgan va dinamikasi tekshirilgan.

[3] ishda tasodifiy parametrik qo‘zg‘alishlar ta’siridagi gisterezis tipidagi elastik dissipativ xarakteristikali sterjenning suyuqlik bo‘g‘inli dinamik so‘ndirgich bilan birgalikdagi chiziqlimas tebranishlari dinamikasi va ustuvorligi masalasi qaralgan. Stoxastik o‘rtalashtirish usulidan foydalangan holda harakat differensial tenglamalar sistemasi Ito tenglamalar sistemasi ko‘rinishiga keltirilgan, tahlil qilingan.

[4] ishda analitik va sonli tahlillar orqali sterjenning bo‘ylama tebranishlari ustuvorligi o‘rganilgan. Sterjenning bo‘ylama tebranishlari ustuvorlik sohasining o‘zgarishida tebranish chastotasi va uzunligi statik holatga qaraganda sezilarli darajada farq qilishi aniqlangan.

[5] maqolada qovushqoq elastik xarakteristikali Releу va Eyler-Bernulli sterjenlarining ko‘ndalang tebranishlari o‘rganilgan. Bundan tashqari, sterjenning ustuvorlik chegaralariga inersiya faktorining ta’siri aniqlangan. Natijalar shuni ko‘rsatadiki, zichlik gradiyenti parametrining pasayishi va elastik modul gradiyenti parametrining oshishi xususiy chastotalarni oshirishi va sterjenning ustuvorlik chegarasini kengaytirishi ko‘rsatilgan. Zichlik va elastik modul gradiyentlari sterjenning tebranma harakatida teskari proportsional ekanligi aniqlangan.

[6-7] ishlarda o‘q bo‘yicha harakatlanuvchi qovushqoq elastik sterjenning chiziqli bo‘lmagan past va yuqori chastotali tebranishlarida amplituda-chastota xarakteristikasi va ustuvorlik chegarasi tahlil qilingan va xususiy hollar bilan taqqoslangan. O‘q bo‘yicha harakatlanuvchi qovushqoq elastik sterjenning ko‘ndalang tebranishini ifodalovchi tenglama chiziqli bo‘lmagan xususiy hosilali differensial tenglama orqali aniqlangan.

[8] ishda tasodifiy qo‘zg‘alishlar ta’siridagi, chiziqli bo‘lmagan, erkinlik darajasi soni ko‘p bo‘lgan Gamilton sistemalarining asimptotik ustuvorligini baholashning taqribiy usuli keltirilgan. Stoxastik o‘rtalashtirish va linearizatsiyalashtirish usullari yordamida Gamilton tenglamalarining eng katta Lyapunov eksponentasi uchun taxminiy formula olingan. Turli ta’sodifiy ta’sirlar uchun asimptotik ustuvorlikning zaruriy va etarlilik shartlari aniqlangan. Tavsiyalar ishlab chiqilgan.

[9] ishda tashqi pulsli shamol ta’siridagi taqsimlangan parametrli sistemaning parametrik tebranishlari ustuvorligi o‘rganilgan. Muhandislik amaliyotida avtomobil yoki samolyot tashqarisidagi havo oqimi doimo pulsatsiyalanuvchi xususiyatni namoyon qilishi, bu elastik strukturaviy komponentlar va tashqi havo oqimini parametrik qo‘zg‘alish sistemasiga aylantirishi ko‘rsatilgan.

[10–11] ishlarda pozitsion kuchlar ta’siridagi taqsimlangan chiziqli mexanik sistemaning tebranishlari qaralgan. Dekompozitsiya usulidan foydalanib, ikkinchi tartibli differentsial tenglamalarning yechimining ustuvorlini tahlil qilish muammosi hal qilinishi mumkin bo‘lgan shartlari olingan. Dekompozitsiya uchun to‘g‘ridan to‘g‘ri Lyapunov usuli qo‘llanilgan.

Ushbu ishda gisterezis tipidagi elastik dissipativ xarakteristikali sterjenning tasodifiy parametrik tebranishlari ustuvorligi masalasini qaraymiz. Bunda sterjen materialining dissipativlik xossalarini ifodalovchi bir qiymatli bo‘lmagan chiziqlimas funksiya statistik chiziqlilashtirish usuli yordamida chiziqli kompleks funksiya bilan almashtirilib tenglamada hisobga olingan [1].

Sterjenning matematik modeli. Sterjenning harakat differensial tenglamasini quyidagicha yozamiz:

$$\ddot{q}_i + \omega_{01}^2(1 + (-v_1 + iv_2 sign\omega)R)q_i = -\omega_{01}^2\xi_0(t)q_i, \tag{1}$$

bu yerda:

$$R = \mathrm{C}_0 + \frac{3EI}{\omega_{01}^2 m d_1}\sum_{j_1=1}^{S_1} C_{j_1}\sigma_{ia}^{j_1}\frac{h^{j_1}}{2^{j_1}(j_1+3)}G_{j_1}; \tag{2}$$

$$G_{j_1} = \int_0^l u_i(x)\frac{\partial^2}{\partial x^2}\left(u_i''(x)|u_i''(x)|^{j_1}\right)dx;\ u_i''(x) = \frac{\partial^2 u_i(x)}{\partial x^2};\ \xi_0(t) = \lambda_*\xi_{0*}(t);$$

$$\lambda_* = -\frac{d_2}{d_1};\ d_1 = \int_0^l u_i^2(x)dx;\ d_2 = \int_0^l u_i(x)dx;$$

m – sterjenning uzunlik birligidagi massasi; E – elastiklik moduli; I –enersiya momenti; $\xi_0(t)$ – statsionar normal tasodifiy jarayonni ifodalovchi o‘zgaruvchan miqdor; ω_{01} – sterjenning xususiy chastotasi; h – sterjen balandligi; ω – tebranishlar chastotasi; l – sterjenning uzunligi; $i^2 = -1$; v_1, v_2 –sterjen materialining dissipativlik xossalarini ifodalovchi chiziqlilashtirish koeffitsiyentlari; x – sterjenning ko‘chish koordinatasi; $q_i(t)$ –vaqtga bog‘liq funksiya; $\mathrm{C}_0, \mathrm{C}_1, \ldots \mathrm{C}_{S_1}$ –lar sterjen materialining dissipativ xarakteristi-

kalarini ifodalovchi gisterezis sirtmog‘idan aniqlanadigan koeffitsiyentlar; σ_{ia} – sterjen ko‘chishlarining o‘rtacha kvadratik qiymatining absolut qiymati; $u_i(x)$ – sterjenning xususiy tebranish formasi.

(1) differensial tenglamaning yechimini quyidagi ko‘rinishda izlaymiz:

$$q_i(t) = A(t)e^{i\omega t} + B(t)e^{-i\omega t}, \tag{3}$$

bu yerda: $A(t), B(t)$lar sekin o‘zgaruvchi funksiyalar hisoblanadi va ampilitudaviy qiyma $\langle q_{ia}\rangle = 2\sqrt{\langle A(t)\rangle\langle B(t)\rangle}$ shartni qanoatlantiradi.

(3) yechimini va undan olingan ikkinchi tartibli hosilani (1) differensial tenglamaga qo‘yamiz va quyidagi birinchi tartibli differensial tenglamalar sistemasini hosil qilamiz:

$$\begin{aligned}\dot{A} = \frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2(1 + (-v_1 + iv_2)\varepsilon R)\right)\left(A + Be^{-2i\omega t}\right) - \\ -\frac{1}{2i\omega}\left(\omega_{01}^2\xi_0(t)\right)\left(A + Be^{-2i\omega t}\right); \\ \dot{B} = -\frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2(1 + (-v_1 - iv_2)\varepsilon R)\right)\left(Ae^{2i\omega t} + B\right) + \\ +\frac{1}{2i\omega}\left(\omega_{01}^2\xi_0(t)\right)\left(Ae^{2i\omega t} + B\right).\end{aligned} \tag{4}$$

(4) differensial tenglamalar sistemasini Ito tenglamalar sistemasiga keltirish maqsadida, dastlab, bu sistemani stoxastik o‘rtalashtiramiz. Bunda (4) differensial tenglamalar sistemasini quyidagi ko‘rinishda yozamiz:

$$\dot{X}_s(t) = f_s(X,t) + \sum_{r=1}^{2} G_{sr}(X,t)\xi_{0r}(t)\,,(s = 1,2) \tag{5}$$

bu yerda:

$$\dot{X}_1 = \dot{A};\ \dot{X}_2 = \dot{B}; \dot{X}_1 = f_1 + G_{11}\xi_0(t);\ \dot{X}_2 = f_2 + G_{22}\xi_0(t);$$

$$f_1(X,t) = \frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2(1 + (-v_1 + iv_2)R)\right)A +$$

$$+\frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2(1 + (-v_1 - iv_2)R)\right)Be^{-2i\omega t};\ G_{11} = -\frac{\omega_{01}^2}{2i\omega}\left(A + Be^{-2i\omega t}\right);$$

$$f_2(X,t) = -\frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2(1 + (-v_1 - iv_2)R)\right)B -$$

$$-\frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2(1 + (-v_1 + iv_2)\varepsilon R)\right)Ae^{2i\omega t}; G_{22} = \frac{\omega_{01}^2}{2i\omega}\left(Ae^{2i\omega t} + B\right).$$

(5) differensial tenglamalar sitemasi uchun quyidagi belgilashlarni kiritamiz:

$$f_1 = p_1 A + p_2 Be^{-2i\omega t};\ p_1 = \frac{1}{2i\omega}(\omega^2 - \omega_{01}^2(1 + (-v_1 + iv_2)R));$$

$$f_2 = p_2 B - p_1 Ae^{2i\omega t};\ p_2 = \frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2(1 + (-v_1 - iv_2)R)\right); p_3 = \frac{\omega_{01}^2}{2i\omega}.$$

U holda (5) differensial tenglamalar sitemasi quyidagicha bo‘ladi:

$$\begin{aligned}\dot{X}_1 = p_1 A + p_2 Be^{-2i\omega t} + \xi_0(t)(-p_3 A - p_3 Be^{-2i\omega t}); \\ \dot{X}_2 = p_2 B - p_1 Ae^{2i\omega t} - \xi_0(t)(p_3\, Ae^{2i\omega t} + p_3 B).\end{aligned} \tag{6}$$

(6) differensial tenglamalar sistemasini stoxastik o‘rtalashtiramiz.

(6) differensial tenglamalarda $\xi_{0r}(t) = \xi_{0*}(t)$ statsionar normal tasodifiy jarayonni ifodalashini hisobga olsak, $\langle d\xi_{0r}(t)\rangle$=$d\langle\xi_{0r}(t)\rangle = d\langle\xi_{0*}(t)\rangle = 0$ bo‘ladi va natijada bu tenglamalar quyidagi ko‘rinishni oladi:

$$\begin{aligned}\frac{d\langle A\rangle}{dt} = \left(p_1 + \frac{\pi}{2}p_3^2\big(S(0) - \psi(2\omega)\big)\right)\langle A\rangle; \\ \frac{d\langle B\rangle}{dt} = \left(p_2 - \frac{\pi}{2}p_3^2\big(S(2\omega) - S(0)\big)\right)\langle B\rangle.\end{aligned} \tag{7}$$

bu yerda $S(0), S(2\omega), \psi(2\omega)$ – $\xi_{0*}(t)$ statsionar normal tasodifiy jarayonning spektral zichliklari [3;48–49-b.].

Ustuvorlik shartlarini aniqlash. (7) differensial tenglamalar sistemasining yechimini quyidagicha izlaymiz:

$$A = A_0(t)e^{-\lambda t};\ B = B_0(t)e^{-\lambda t}, \tag{8}$$

bu yerda A_0, B_0 – sterjenning tasodifiy parametrik qo‘zg‘alishlarining ampilitudaviy qiymatlari; λ –xarakteristik son.

(8) yechimlarni (7) tenglamalar sistemasiga qo‘ysak, xarakteristik tenglamaga ega bo‘lamiz. Bu xaraktrestik tenglamaning ildizlari quyidagicha bo‘ladi:

$$\lambda_1 = -\frac{\omega_{01}^2}{2\omega}\left(v_2 R + \frac{\pi\omega_{01}^2}{4\omega}\big(S(0) - \psi(2\omega)\big)\right) + i\left(\frac{\omega_{01}^2}{2\omega}(1 - v_1 R) - \frac{\omega}{2}\right);$$
$$\lambda_2 = \frac{\omega_{01}^2}{2\omega}\left(v_2 R + \frac{\pi\omega_{01}^2}{4\omega}\big(S(2\omega) - S(0)\big)\right) + i\left(\frac{\omega_{01}^2}{2\omega}(1 - v_1 R) - \frac{\omega}{2}\right). \quad (9)$$

Aniqlangan xarakteristik tenglamaning ildizlari qaralayotgan tebranishlardan himoyalanuvchi sterjenning o‘rtacha kvadratik qitmatlarini va ustuvorligini tekshirish imkonini beradi.

(9) ildizlardagi R hadni (2) ifodaga asosan, $S_1 = 2$ bo‘lganda yozamiz. Ustuvorlik nazariyasiga ko‘ra qaralayotgan harakat asimptotik ustuvor bo‘lishi uchun xarakteristik tenglamalar ildizlarining haqiqiy qismi manfiy bo‘lishi yetarli.

$$-\frac{\pi\omega_{01}^2}{4\omega v_2}[S(0) - \psi(2\omega)] < C_0 + \frac{3EIh}{8\omega_{01}^2 m d_1} C_1\sigma_{ia} +$$
$$+\frac{3EIh^2}{20\omega_{01}^2 m d_1} C_2\sigma_{ia}^2 < -\frac{\pi\omega_{01}^2}{4\omega v_2}[S(2\omega) - S(0)]. \quad (10)$$

Olingan (10) qo‘sh tengsizlik gisterezis tipidagi elastik dissipativ xarakteristikali sterjenning tasodifiy parametrik tebranishlarida ustuvorlik sharti hisoblanadi.

Xulosa. Gisterezis tipidagi elastik dissipativ xarakteristikali sterjenning tasodifiy parametrik ko‘ndalang tebranishlari ustuvorlik shartlari Ito usuli yordamida aniqlangan. Aniqlangan ustuvorlik shartlari parametrlarning turli qiymatlari va spektral zichlikning turli xil ko‘rinishlardagi ifodalari uchun sterjenning tasodifiy parametrik ko‘ndalang tebranishlari ustuvorligini tekshirish imkonini beradi.

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