## FIZIKA-MATEMATIKA

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# MATRITSAVIY YUQORI YARIM TEKISLIK AVTOMORFIZMLARI XOSSALARI

***Annotatsiya.*** *Matritsaviy sohalarni o‘rganish ko‘p o‘lchovli kompleks analizda muhim hisoblanadi. Ushbu maqolada matritsaviy yuqori yarim tekislikning avtomorfizmlari xossalari o‘rganilgan bo‘lib, avtomorfizmlar to‘plamining kompozitsiya amaliga nisbatan gruppa tashkil etishi keltirilgan.*

***Kalit so‘zlar:*** *matritsaviy yuqori yarim tekislik, matritsaviy birlik doira, avtomorfizm, unitar matritsa, ermit matritsa, ostav, kelajak trubasi, kompozitsiya amali, gruppa.*

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# СВОЙСТВА АВТОМОРФИЗМОВ МАТРИЧНОЙ ВЕРХНЕЙ ПОЛУПЛОСКОСТИ

***Аннотация.*** *Изучение матричных областей имеет важное значение в многомерном комплексном анализе. В данной статье изучены свойства автоморфизмов матричной верхней полуплоскости и показано, что множество автоморфизмов образует группу относительно операции композиции.*

***Ключевые слова:*** *татричная верхняя полуплоскость, татричный единичный круг, автоморфизм, унитарная матрица, эрмитова матрица, остов, будущая труба, операция композиции, группа.*

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# PROPERTIES OF AUTOMORPHISMS OF THE MATRIX UPPER HALF-PLANE

***Annotation.*** *The study of matrix domains is important in multidimensional complex analysis. This article studies the properties of automorphisms of the matrix upper half-plane and shows that the set of these automorphisms forms a group under the operation of composition.*

***Keywords:*** *matrix upper half-plane, matrix unit disk, automorphism, unitary matrix, hermitian matrix, sceleton, future tube, composition operation, group.*

**Kirish.** Ma’lumki, bir kompleks o‘zgaruvchili funksiyalar nazariyasida ko‘p hollarda $\mathbb{U}=\{z\in\mathbb{C}:|z|<1\}$ birlik doiradagi funksiyalar nazariyasi o‘rganiladi, bu klassik Riman teoremasini tekislikdagi ko‘pgina simmetrik sohalar birlik doiraga bigolomorf ekvivalent bilan izohlanadi. Shu sababli birlik doira va unga konform ekvivalent sohalarning avtomorfizmlari kompleks analizda muhim rol o‘ynaydi. Ko‘p o‘lchovli kompleks analizda bu, umuman olganda, o‘rinli bo‘lmagani uchun o‘zaro bigolomorf funksiyalar sinfi muhim sinflardan hisoblanadi. Bunda, ko‘pincha, chegaralanmagan sohalarda golomorf funksiyalarning xossalarini o‘rganishda, bu sohalarning chegaralangan sohalar bilan o‘zaro bigolomorf ekvivalentligi keng tadqiq qilinadi.

XX asrning oxiri va XXI asr boshlarida V.S.Vladimirov, A.G.Sergeev, G.Xenkin, S.G.Gindikin, Xio -Ming, A.M.Kytmanov, S.G.Myslives, T.N.Nikitina va boshqa olimlar tomonidan matritsaviy sohalarda golomorf funksiyalarning xossalarini o‘rganish ishlari davom ettirildi. Ushbu ilmiy izlanishlar ayniqsa, chegaralanmagan sohalarda golomorf davom qildirish nazariyasining asoslarini yaratish va ularni chegaralangan sohalar bilan bigolomorf ekvivalentlik orqali tadqiq etish yo‘nalishida olib borildi.

Ko‘p o‘lchovli kompleks analizning integral formulalari nazariyasi va ularning amaliy qo‘llanilishi bo‘yicha yurtimizda G.Xudoyberganov, S.Kosbergenov, B.A.Shaimkulov, B.Otemuratov, B.Prenov, B. Kurbanov va boshqa ilmiy tadqiqotchilar faoliyat yuritmoqdalar. Ushbu tadqiqotlar jarayonida fazoviy va matritsaviy sohalar doirasida muhim ilmiy natijalarga erishilgan. Shunga qaramay, bu yo‘nalishda hali to‘liq yechim topmagan bir qator muammolar mavjud bo‘lib, ularni hal etish dolzarb vazifalardan biri hisoblanadi.

Mazkur maqolada matritsaviy yuqori yarim tekislikning avtomorfizmlari tahlil qilinadi hamda ushbu avtomorfizmlar to‘plamining kompozitsiya amali bo‘yicha gruppa hosil qilishi isbotlanadi.

$m^2$ ta kompleks o‘zgaruvchili $\mathbb{C}^{m^2}$ fazoni qaraylik. Bazi masalalarda bu fazoni $Z$ nuқtaларини $Z=(z_{ij})_{i,j=1}^{m}$ ko‘rinishida yozish, ya’ni, $[m\times m]$-tartibli matritsalar fazosi sifatida qarash qulay. $\mathbb{C}^{m^2}$ fazo nuqtalarini bunday ko‘rinishda yozishdan hosil bo‘lgan fazoni $\mathbb{C}[m\times m]$ orqali belgilaymiz.

Elementlari $\mathbb{C}$ kompleks sonlardan iboart bo‘lgan, $m$-tartibli

$$Z=\begin{pmatrix} z_{11} & z_{12} & \dots & z_{1m} \\ z_{21} & z_{22} & \dots & z_{2m} \\ \dots & \dots & \dots & \dots \\ z_{m1} & z_{m2} & \dots & z_{mm} \end{pmatrix}$$

Kvadrat matritsaning elelementlarini $\mathbb{C}^{m^2}$ fazoning $z$ nuqtasi sifatida qaraymiz:

$$z=(z_{11},\dots,z_{21},\dots,z_{m1},\dots,z_{mm})\in\mathbb{C}^{m^2}$$

Bu fazodagi ushbu

$$\tau=\{\mathbb{C}[m\times m]: ZZ^*<I\}$$

sohani matritsaviy birlik doira deb ataymiz. Agar $m=1$ bo‘lsa, matritsaviy birlik doira $\mathbb{C}$ kompleks tekislikdagi oddiy ma’nodagi birlik doira bilan ustma-ust tushadi.

$$\tau=\{\mathbb{C}[1\times 1]: ZZ^*<I\}=\{\mathbb{C}[1\times 1]: z_{11}z_{11}{}^*<1\}=\{\mathbb{C}[1\times 1]: |z_{11}|^2<1\}$$

Quyidagi

$$H=\{Z\in\mathbb{C}[m\times m]: \operatorname{Im}Z>0\}$$

ko‘rinishdagi sohaga umumlashgan matritsaviy yuqori yarim tekislik (qarang [1]) deyiladi . Bu yerda $\operatorname{Im}Z=\dfrac{1}{2i}(Z-Z^*)$ ifodaga teng. Bu $\operatorname{Im}Z>0$ matritsaning musbat aniqlanganligini, uning barcha xos sonlarining musbat ekanligini bildiradi.

Ushbu

$$S(H)=\{Z\in\mathbb{C}[m\times m]: \operatorname{Im}Z=0\}$$

to‘plamga umumlashgan matritsaviy yuqori yarim tekislikning ostovi deyiladi. Demak, $S(H)$ barcha Ermit matritsalardan iborat ekan (Ermit matritsa (qarang[1]) deb $A=A^*$ o‘rinli bo‘ladigan matritsalarga aytiladi).

Agar $m=1$ bo‘lsa, $Z=z_{11}$ bo‘lib,

$$\operatorname{Im}Z=\frac{1}{2i}(z_{11}-z_{11}{}^*)=\frac{1}{2i}\cdot 2iy=y>0$$

bo‘ladi. Bu esa uning $\mathbb{C}$ dagi yuqori yarim tekislik ekanligini ifodalaydi.

Agar $m=2$ bo‘lsa, soha quyidagi ko‘rinishga keladi:

$$H=\left\{Z\in\mathbb{C}[2\times 2]: \begin{pmatrix} \operatorname{Im}z_{11} & \dfrac{z_{12}-\overline{z}_{21}}{2i} \\ \dfrac{z_{21}-\overline{z}_{12}}{2i} & \operatorname{Im}z_{22} \end{pmatrix}>0\right\}.$$

Agar uni soddalashtirsak,

$$H=\left\{\operatorname{Im}z_{11}>0, \operatorname{Im}z_{11}\operatorname{Im}z_{22}-\frac{1}{4}|z_{12}-\overline{z}_{21}|^2>0\right\},$$

bu sohaning chegarasi

$$\partial H=\left\{\operatorname{Im}z_{11}\operatorname{Im}z_{22}=\frac{1}{4}|z_{12}-\overline{z}_{21}|^2\right\},$$

bu yerda ostov haqiqiy 4 o‘lchamli tekislikni ifodalaydi.

$$S(H)=\{\operatorname{Im}z_{11}=\operatorname{Im}z_{22}=0, z_{12}=\overline{z}_{21}\}.$$

takidlash joizki, quyidagi xosmas affin almashtirish,

$$\Phi(Z)=\begin{pmatrix} z_{11}+z_{22} & z_{12}+z_{21} \\ i(z_{21}-z_{12}) & z_{11}-z_{22} \end{pmatrix}$$

sohani quyidagi tengsizlik bilan aniqlanadigan sohaga akslantiradi

$$H=\left\{\operatorname{Im} z_{11}>0,\left(\operatorname{Im} z_{11}\right)^2>\left(\operatorname{Im} z_{12}\right)^2+\left(\operatorname{Im} z_{21}\right)^2+\left(\operatorname{Im} z_{22}\right)^2\right\}.$$

Odatda, bu sohaga kelajak trubasi (qarang [2]) deyiladi.

**Matritsaviy yuqori yarim tekislikning avtomorfizmlari.** Odatda, sodda sohalarning avtomorfizmlari kasr chiziqli akslantirish ko‘rinishida qidiriladi. Tabiiy ravishda

$$W=(AZ+B)\cdot(CZ+D)^{-1} \tag{1}$$

kasr chiziqli matritsaviy funksiyani qaraymiz, bu yerda $A,B,C,D\in\mathbb{C}[m\times m]$ matritsalar. (1) aksllantrishni $A,B,C,D$ parametrlari qanday shartlar bajarilganda, uni $H$ ning avtomorfizmi bo‘lish maslalasini qaraymiz. Aniqlanishga ko‘ra, buning uchun $\operatorname{Im} W$ ni hisoblaymiz:

$$\operatorname{Im} W=\frac{1}{2i}\left(W-W^*\right)=$$

$$=\frac{1}{2i}\left[(AZ+B)\cdot(CZ+D)^{-1}-(Z^*C^*+D^*)^{-1}\cdot(Z^*A^*+B^*)\right]=$$

$$=\frac{1}{2i}(Z^*C^*+D^*)^{-1}\cdot\left[(Z^*C^*+D^*)\cdot(AZ+B)-\right.$$

$$\left.-(Z^*A^*+B^*)\cdot(CZ+D)\right]\cdot(CZ+D)^{-1}=$$

$$=\frac{1}{2i}\left(Z^*C^*+D^*\right)^{-1}\left[Z^*C^*AZ-Z^*A^*CZ+Z^*C^*B-Z^*A^*D+\right.$$

$$\left.+D^*AZ-B^*CZ+D^*B-B^*D\right]\left(CZ+D\right)^{-1}=$$

$$=\frac{1}{2i}(Z^*C^*+D^*)^{-1}\cdot\left[Z^*(C^*A-A^*C)Z+Z^*(C^*B-A^*D)+\right.$$

$$\left.+(D^*A-B^*C)Z+D^*B-B^*D\right]\cdot(CZ+D)^{-1}.$$

tenglikning oxiridan

$$C^*A=A^*C \tag{2}$$

$$D^*B=B^*D \tag{3}$$

$$D^*A-B^*C=I \tag{4}$$

deb shart olsak, (4) tenglikdan $C^*B-A^*D=-I$ tenglikka ega bo‘lamiz. U holda $\operatorname{Im} W$ matritsa quyidagicha ko‘rinish oladi:

$$\operatorname{Im} W=(Z^*C^*+D^*)^{-1}\cdot\operatorname{Im} Z\cdot(CZ+D)^{-1}. \tag{5}$$

Agar $Z$ ni $S(H)$ to‘plamdan oladigan bo‘lsak $\operatorname{Im} Z=0$ bo‘ladi. Bundan esa $\operatorname{Im} W=O$ ($O$ barcha elementlari 0 dan iborat kvadrat matritsa) ekanligi kelib chiqadi. Ya’ni, (1) akslantirish (2), (3), (4) shartlar bajarilganda $H$ sohaning ostovini o‘zida saqlar ekan.

Agar yuqoridagi ifodada $C$ matritsa aynimagan bo‘lsa, u holda (1) akslantirishning birinchi qovusini

$$AZ+B=AC^{-1}CZ+AC^{-1}D+B-AC^{-1}D=$$

$$=AC^{-1}\left(CZ+D\right)+B-AC^{-1}D\Rightarrow$$

$$\Rightarrow W=\left(AZ+B\right)\left(CZ+D\right)^{-1}=AC^{-1}+\left(B-AC^{-1}D\right)\left(CZ+D\right)^{-1}$$

ko‘rinishga almashtirish mumkin, (2) tenglikdan

$$C^*A = A^*C$$
$$A = \left(C^{-1}\right)^* A^*C$$
$$AC^{-1} = \left(C^{-1}\right)^* A^*CC^{-1}$$
$$AC^{-1} = \left(C^{-1}\right)^* A^*$$
$$AC^{-1} = \left(AC^{-1}\right)^*$$

$AC^{-1}$ ning Ermit matritsa ekanligi
(4) tenglikdan esa

$$D^*A - B^*C = I$$
$$A^*D - C^*B = I$$
$$\left(C^{-1}\right)^* A^*D - B = \left(C^{-1}\right)^*$$
$$\left(AC^{-1}\right)^* D - B = \left(C^{-1}\right)^*$$
$$AC^{-1}D - B = \left(C^{-1}\right)^*$$
$$B - AC^{-1}D = -(C^{-1})^*$$

kelib chiqadi. Bu munosabatlardan foydalanib, quyidagi tenglikka ega bo‘lamiz:

$$W = AC^{-1} - (C^{-1})^* \cdot (Z + C^{-1}D)^{-1}C^{-1} \tag{6}$$

va yana

$$A^{-1}B - C^{-1}D = -A^{-1}(C^{-1})^*$$
$$C^{-1}D = A^{-1}B + A^{-1}(C^{-1})^*$$
$$B - AC^{-1}D = -(C^{-1})^*$$
$$C^{-1}D = \left(B^{-1}A\right)^{-1} + \left(C^*A\right)^{-1}$$

$B^{-1}A$ va $C^*A$ matritsalar Ermit bo‘lganligidan $C^{-1}D$ Ermit matritsaligi kelib chiqadi. O‘z navbatida, $C^{-1}D$ Ermit matritsasi ekanligidan $\operatorname{Im}(Z + C^{-1}D) = \operatorname{Im} Z$ bo‘ladi. Natijada barcha $Z \in H$ matritsalar uchun $Z + C^{-1}D \in H$ va $Z + C^{-1}D$ matritsa aynimaganligi kelib chiqadi. U holda (6) ifoda uchun $\operatorname{Im} W$ ni hisoblasak,

$$\operatorname{Im} W = \frac{1}{2i}\left(W - W^*\right) =$$
$$= \frac{1}{2i}\left(AC^{-1} - (C^{-1})^* \cdot (Z + C^{-1}D)^{-1}C^{-1} - \left(AC^{-1}\right)^* + (C^{-1})^* \cdot \left((Z + C^{-1}D)^{-1}\right)^* C^{-1}\right) =$$
$$= \frac{1}{2i}(C^{-1})^*\left(\left((Z + C^{-1}D)^{-1}\right)^* - (Z + C^{-1}D)^{-1}\right)C^{-1} =$$
$$= \frac{1}{2i}(C^{-1})^*\left((Z + C^{-1}D)^{-1}\right)^*\left[(Z + C^{-1}D) - \left(Z + C^{-1}D\right)^*\right](Z + C^{-1}D)^{-1}C^{-1} =$$
$$= \frac{1}{2i}(C^{-1})^*\left((Z + C^{-1}D)^{-1}\right)^*\left[Z + C^{-1}D - Z^* - C^{-1}D\right](Z + C^{-1}D)^{-1}C^{-1} =$$
$$= (C^{-1})^*\left((Z + C^{-1}D)^{-1}\right)^* \operatorname{Im} Z (Z + C^{-1}D)^{-1}C^{-1}.$$

$\operatorname{Im} Z > 0$ bo‘lganligi uchun $\operatorname{Im} W > 0$ bo‘ladi.

Bundan (1) akslantirish $H$ sohada golomorf ekani kelib chiqadi. Bundan tashqari, ko‘rinib turibdiki, u $H$ sohani $H$ sohaga, uning teskarisi esa $C^{-1}D$ ifodani $AC^{-1}$ ifodaga o‘tkazadi. Shunday qilib, (1) akslantirish $H$ sohani o‘ziga akslantiruvchi bigolomorf akslantirish bo‘ladi.

Koeffitsiyentlari (2), (3), (4) shartlarni qanoatlantiruvchi (1) ko‘rinishdagi $W$ bigolomorf akslantirishlar to‘plamini $\Gamma$ orqali belgilaymiz.

**1-teorema.** *H matritsaviy yuqori yarimningning $\Gamma$ avtomorfizmlar to‘plami kompozitsiya amaliga nisbatan gruppa tashkil etadi.*

**Isbot.** Gruppa ta’rifiga ko‘ra, $\Gamma$ to‘plam va kompozitsiya amali uchun quyidagi 3 ta shartning bajarilishini ko‘rsatishimiz kerak.

1) ixtiyoriy $W_1, W_2, W_3 \in \Gamma$ uchun

$$\left(W_1 \circ W_2\right) \circ W_3 = W_1 \circ \left(W_2 \circ W_3\right)$$

tenglik o‘rinli bo‘lishi kerak;

2) shunday $E \in \Gamma$ birlik akslantirish topilishi kerakki, ixtiyoriy $W \in \Gamma$ uchun $W \circ E = E = E \circ W$
tenglik bajarilishi kerak;

3)uchun shunday $W^{-1} \in \Gamma$ teskari akslantirish topilishi kerakki, $W \circ W^{-1} = E = W^{-1} \circ W$
bo‘lishi kerak.

**1-qadam.** Dastlab, birinchi shartning bajarilishini ko‘rstamiz. Buning uchun

$$W_1 = \left(A_1 Z + B_1\right)\left(C_1 Z + D_1\right)^{-1}$$
$$W_2 = \left(A_2 Z + B_2\right)\left(C_2 Z + D_2\right)^{-1}$$

deb olamiz. U holda

$$\left(W_1 \circ W_2\right) \circ W_3 = \left(A_1\left(A_2 Z + B_2\right)\left(C_2 Z + D_2\right)^{-1} + B_1\right) \cdot$$
$$\cdot \left(C_1\left(A_2 Z + B_2\right)\left(C_2 Z + D_2\right)^{-1} + D_2\right)^{-1} \circ W_3 =$$
$$= \left(A_1\left(A_2 W_3 + B_2\right)\left(C_2 W_3 + D_2\right)^{-1} + B_1\right) \cdot$$
$$\cdot \left(C_1\left(A_2 W_3 + B_2\right)\left(C_2 W_3 + D_2\right)^{-1} + D_2\right)^{-1},$$

boshqa tomondan esa

$$W_1 \circ \left(W_2 \circ W_3\right) = W_1 \circ \left(\left(A_2 W_3 + B_2\right)\left(C_2 W_3 + D_2\right)^{-1}\right) =$$
$$= \left(A_1\left(A_2 W_3 + B_2\right)\left(C_2 W_3 + D_2\right)^{-1} + B_1\right) \cdot$$
$$\cdot \left(C_1\left(A_2 W_3 + B_2\right)\left(C_2 W_3 + D_2\right)^{-1} + D_2\right)^{-1}$$

ko‘rinishga keladi. Yuqoridagi oxirgi ikki ifodadan ko‘rinadiki, chap va o‘ng tomonlar aynan teng. bu esa 1-shartning bajarilishini ko‘rsatadi.

**2-qadam** (1) dagi akslantirishda $A = I,\ B = O, C = \mathrm{O}, D = I$ deb olsak, $W = Z$ ko‘rinishga keladi va $A, B, C, D$ lar yuqoridagi (2), (3), (4) shartlarni ham bajaradi.

Ayniy birlik akslantirish sifatida $E = Z$ deb olsak $E \in \Gamma$ bo‘ladi va

$$W \circ E = W \circ Z = W$$
$$E \circ W = Z \circ W = W$$

shartlar bajariladi.

**3-qadam.** $W$ ga teskari akslantirish sifatida $W^{-1}=\left(A_1Z+B_1\right)\left(C_1Z+D_1\right)$ akslantirishni qaraylik. $A_1,B_1,C_1,D_1$ lar uchun qanday shart bajarilganda, $W\circ W^{-1}=E=W^{-1}\circ W$ tenglikning o‘rinli bo‘lishi va $W^{-1}\in\Gamma$ bo‘lishini ko‘rsatishimiz kerak.

$$W\circ W^{-1}=\left(AW^{-1}+B\right)\left(CW^{-1}+D\right)^{-1}=$$

$$=\left(A\left(A_1Z+B_1\right)\left(C_1Z+D_1\right)^{-1}+B\right)\left(C\left(A_1Z+B_1\right)\left(C_1Z+D_1\right)^{-1}+D\right)^{-1}=$$

$$=\left\{A\left(A_1Z+B_1\right)+B\left(C_1Z+D_1\right)\right\}\left(C_1Z+D_1\right)^{-1}\cdot$$

$$\cdot\left(C_1Z+D_1\right)\left\{C\left(A_1Z+B_1\right)+D\left(C_1Z+D_1\right)\right\}^{-1}=$$

$$=\left\{\left(AA_1+BC_1\right)Z+AB_1+BD_1\right\}\left\{\left(CA_1+DC_1\right)Z+CB_1+DD_1\right\}^{-1}.$$

Bu tenglikning oxiridan

$$\begin{aligned}AA_1+BC_1&=I\\AB_1+BD_1&=\mathrm{O}\\CA_1+DC_1&=\mathrm{O}\\CB_1+DD_1&=I\end{aligned}\qquad(7)$$

deb olsak, u holda

$$W\circ W^{-1}=Z$$

bo‘ladi. Endi $W^{-1}\in\Gamma$ ekanligini ko‘rsatishimiz kerak. Buning uchun $A_1,B_1,C_1,D_1$ koeffitsiyentlar uchun (2), (3), (4) shartlarning bajarilishi ko‘rsatishimiz lozim. Dastlab, $A_1,B_1,C_1,D_1$ koeffitsiyentlarni topib olaylik. (7) ning birinchi va uchinchi tengliklaridan

$$\begin{cases}AA_1+BC_1=I\\CA_1+DC_1=\mathrm{O}\end{cases}$$

$$\begin{cases}B^{-1}AA_1+C_1=B^{-1}\\D^{-1}CA_1+C_1=\mathrm{O}\end{cases}$$

$$\left(B^{-1}A-D^{-1}C\right)A_1=B^{-1}\ \Rightarrow\ A_1=\left(B^{-1}A-D^{-1}C\right)^{-1}B^{-1}$$

$$C_1=-D^{-1}CA_1=-D^{-1}C\left(B^{-1}A-D^{-1}C\right)^{-1}B^{-1}$$

bo‘ladi. (7) ifodaning ikkinchi va to‘rtinchi tengliklaridan esa

$$\begin{cases}CB_1+DD_1=I\\AB_1+BD_1=\mathrm{O}\end{cases}$$

$$\begin{cases}D^{-1}CB_1+D_1=D^{-1}\\B^{-1}AB_1+D_1=\mathrm{O}\end{cases}$$

$$\left(D^{-1}C-B^{-1}A\right)B_1=D^{-1}\ \Rightarrow B_1=\left(D^{-1}C-B^{-1}A\right)D^{-1}$$

$$D_1=-B^{-1}AB_1=-B^{-1}A\left(D^{-1}C-B^{-1}A\right)D^{-1}$$

ekanligi kelib chiqadi.

$A_1,B_1,C_1,D_1$ lar uchun (2) shartning bajarilishini ko‘rsatamiz:

$$C_1^* A_1 = -\left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* C^* \left(D^{-1}\right)^* \left(B^{-1}A - D^{-1}C\right)^{-1} B^{-1}$$

$$A_1^* C_1 = -\left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* D^{-1}C \left(B^{-1}A - D^{-1}C\right)^{-1} B^{-1}.$$

Bizda $C^{-1}D$ matritsaning Ermit ekanligi bor. Ermit matritsaning "Ermit matritsaning teskarisi yana Ermit matritsa bo'ladi" degan xossasiga ko'ra, $\left(C^{-1}D\right)^{-1} = D^{-1}C$ ham Ermit bo'ladi. U holda

$$C_1^* A_1 = -\left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* C^* \left(D^{-1}\right)^* \left(B^{-1}A - D^{-1}C\right)^{-1} B^{-1} =$$

$$= -\left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* \left(D^{-1}C\right)^* \left(B^{-1}A - D^{-1}C\right)^{-1} B^{-1} =$$

$$= -\left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* D^{-1}C \left(B^{-1}A - D^{-1}C\right)^{-1} B^{-1}.$$

Bundan quyidagi shart kelib chiqadi:

$$C_1^* A_1 = A_1^* C_1$$

Demak, $A_1, B_1, C_1, D_1$ lar uchun (2) shart bajarilar ekan.

Endi $A_1, B_1, C_1, D_1$ matritsalar uchun (4) shartning bajarilishini ko'rsatamiz:

$$C_1^* B_1 - A_1^* D_1 = -\left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* C^* \left(D^{-1}\right)^* \left(B^{-1}A - D^{-1}C\right)^{-1} D^{-1} +$$

$$+\left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* B^{-1}A \left(B^{-1}A - D^{-1}C\right)^{-1} D^{-1} =$$

$$= \left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* \left[-C^* \left(D^{-1}\right)^* + B^{-1}A\right] \left(B^{-1}A - D^{-1}C\right)^{-1} D^{-1}$$

(3) va (4) tengliklardan foydalanib,

$$D^*A - B^*C = I$$

$$A = \left(D^{-1}\right)^* B^*C + \left(D^{-1}\right)^*$$

$$B^{-1}A = B^{-1}\left(D^{-1}\right)^* B^*C + B^{-1}\left(D^{-1}\right)^* = \left(D^*B\right)^{-1} B^*C + \left(D^*B\right)^{-1} =$$

$$= D^{-1}\left(B^{-1}\right)^* B^*C + \left(D^*B\right)^{-1} = D^{-1}C + \left(D^*B\right)^{-1}.$$

Bizda $D^{-1}C$ va $\left(D^*B\right)^{-1}$ matritsalarning Ermitligi bor. Ularning yig'indisi ham yana ermit bo'ladi, ya'ni $B^{-1}A$ matritsa Ermit. U holda

$$C_1^* B_1 - A_1^* D_1 = \left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* \left[-C^* \left(D^{-1}\right)^* + B^{-1}A\right] \left(B^{-1}A - D^{-1}C\right)^{-1} D^{-1} =$$

$$= \left(B^{-1}\right)^* \left(\left(B^{-1}A - D^{-1}C\right)^{-1}\right)^* \left[B^{-1}A - D^{-1}C\right]^* \left(B^{-1}A - D^{-1}C\right)^{-1} D^{-1} =$$

$$= \left(B^{-1}\right)^* \left(B^{-1}A - D^{-1}C\right)^{-1} D^{-1} = \left(B^{-1}\right)^* \left(DB^{-1}A - C\right)^{-1} =$$

$$= \left(DB^{-1}AB^* - CB^*\right)^{-1} = \left(D\left(B^{-1}A\right)^* B^* - CB^*\right)^{-1} = \left(DA^*\left(B^{-1}\right)^* B^* - CB^*\right)^{-1} =$$

$$= \left(DA^* - CB^*\right)^{-1}$$

Bu esa $A_1, B_1, C_1, D_1$ lar uchun (4) shartning bajarilishini ko'rsatadi.

Endi $A_1, B_1, C_1, D_1$ lar uchun (3) shartning bajarilishini ko‘rsatamiz:

$$D_1^* B_1 = -\left(D^{-1}\right)^* \left(\left(D^{-1}C - B^{-1}A\right)^{-1}\right)^* A^* \left(B^{-1}\right)^* \left(D^{-1}C - B^{-1}A\right)^{-1} D^{-1}$$

$$B_1^* D_1 = -\left(D^{-1}\right)^* \left(\left(D^{-1}C - B^{-1}A\right)^{-1}\right)^* B^{-1}A\left(D^{-1}C - B^{-1}A\right)^{-1} D^{-1}$$

$B^{-1}A$ ning Ermit ekanligidan, $A^*\left(B^{-1}\right)^* = \left(B^{-1}A\right)^* = B^{-1}A$ bo‘lishi kelib chiqadi. Bundan esa $D_1^* B_1 = B_1^* D_1$ ekanligi kelib chiqadi, ya’ni $A_1, B_1, C_1, D_1$ lar uchun (3) ham bajarilar ekan. Bundan xulosa qila olamizki, $W^{-1} \in \Gamma$ ekan. Teorema isbotlandi.

**Xulosa.** Agar $m = 1$ bo‘lsa, $Z = z,\ A = a,\ B = b,\ C = c,\ D = d$ bo‘lib, (1) akslantirish

$$w = \frac{az + b}{cz + d}$$

ko‘rinishga (2), (3), (4) shartlar esa

$$\overline{c}a = \overline{a}c$$
$$\overline{d}b = \overline{b}d$$
$$\overline{d}a - \overline{b}c = 1$$

ko‘rinishga keladi. Bu esa uning xususiy holda, $\mathbb{C}$ dagi yuqori yarim tekislikning avtomorfizmi bilan ustma-ust tushadi. Demak, biz topgan (1) akslantirish umumiy holda, matritsaviy yuqori yarim tekislikning avtomorfizmi bo‘lar ekan.

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## PIKOSEKUND IMPULSLI Nd:YAG LAZER YORDAMIDA ALUMINIY YUZASIDA KRATER HOSIL BO‘LISHI VA PLAZMANING SPEKTRAL XUSUSIYATLARI

***Annotatsiya.*** *Ushbu ilmiy ishda pikosekund impulsli Nd:YAG lazer (to‘lqin uzunligi 1064 nm, impuls davomiyligi 28 ps) yordamida yuqori intensivlikdagi lazer nurlanishining aluminiy (Al) folga yuzasiga ta’siri o‘rganildi. Tadqiqotda lazer intensivligi 5.86× $10^{11}$ W/cm$^2$ diapazonida bo‘lib, lazer ablatsiyasi jarayoni va hosil bo‘lgan plazmaning fizik xususiyatlari mikroskopik krater tasvirlari hamda optik emissiya spektroskopiyasi asosida tahlil qilindi.*

***Kalit so‘zlar:*** *pikosekundli lazer, lazer induksiyalangan plazma, aluminiy folga, krater, elektron zichligi.*

## ФОРМИРОВАНИЕ КРАТЕРОВ И СПЕКТРАЛЬНЫЕ ХАРАКТЕРИСТИКИ ПЛАЗМЫ НА ПОВЕРХНОСТИ АЛЮМИНИЯ ПОД ДЕЙСТВИЕМ ПИКОСЕКУНДНЫХ ИМПУЛЬСОВ Nd: YAG ЛАЗЕРА

***Аннотация.*** *В данной научной работе исследовано воздействие высокоинтенсивного лазерного излучения пикосекундного Nd:YAG лазера (длина волны 1064 нм, длительность импульса 28 пс)*