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RADIATION PRESSURE FROM MAXWELL’S EQUATIONS: A STEP-BY-STEP DERIVATION

Annotation. *We derive, directly from Maxwell's equations, that electromagnetic waves carry momentum and exert mechanical pressure on matter. Starting from Poynting's theorem (energy conservation) we introduce the electromagnetic momentum density and the Maxwell stress tensor via the momentum-balance law. Evaluating the stress on a surface for a plane wave yields the standard results: for normal incidence, a perfectly absorbing surface experiences a pressure ($p = I/c$), while a perfectly reflecting surface experiences ($p = 2I/c$), where I is the intensity and c is the speed of light.*

Keywords: *radiation pressure, intensity, Maxwell's equation, stress tensor.*

MAKSVELL TENGLAMALARI ASOSIDA NURLANISH BOSIMI: BOSQICHMA-BOSQICH KELTIRIB CHIQARISH

Annotatsiya. *Biz Maksvell tenglamalaridan bevosita elektromagnit to‘lqinlar impuls tashishi va moddaga mexanik bosim ko‘rsatishi xulosasini keltirib chiqaramiz. Poynting teoremasidan (energiya saqlanish qonuni) boshlab, impuls balansi qonuni yordamida elektromagnit impuls zichligi va Maksvell kuchlanish tenzori tushunchalarini kiritamiz. Yassi to‘lqin uchun sirtga ta’sir etuvchi kuchlanishni tahlil qilish standart natijalarni beradi: to‘lqin tik tushganda, mukammal yutuvchi sirtga p = I/c bosim, mukammal qaytaruvchi sirtga esa p = 2I/c bosim ta’sir qiladi (bu yerda I - intensivlik, c - yorug‘lik tezligi).*

Kalit so‘zlar: *nurlanish bosimi, intensivlik, Maksvell tenglamasi, kuchlanish tenzori.*

ДАВЛЕНИЕ ИЗЛУЧЕНИЯ НА ОСНОВЕ УРАВНЕНИЙ МАКСВЕЛЛА: ПОШАГОВЫЙ ВЫВОД

Аннотация. *Исходя непосредственно из уравнений Максвелла, мы показываем, что электромагнитные волны переносят импульс и оказывают механическое давление на вещество. Опираясь на теорему Пойнтинга (закон сохранения энергии), мы вводим понятие плотности электромагнитного импульса и тензор напряжений Максвелла на основе закона баланса импульса. Расчет давления, оказываемого плоской волной на поверхность, приводит к стандартным результатам: при нормальном падении волны на идеально поглощающую поверхность давление составляет p = I/c, а на идеально отражающую – p = 2I/c (где I – интенсивность, а c - скорость света).*

Ключевые слова: *давление излучения, интенсивность, уравнение Максвелла, тензор напряжений.*

Introduction. Radiation pressure, the mechanical force exerted by electromagnetic radiation, is a fundamental prediction of Maxwell's theory of electrodynamics. From the earliest formulations of the electromagnetic field equations by Maxwell [1,2] and the introduction of the Poynting vector [3], the intimate relationship between energy transport, momentum density, and mechanical effects of light has been recognized. These theoretical foundations were later supported by classical experiments such as Lebedev's measurement of light pressure [4] and the pioneering work of Nichols and Hull [5], which confirmed the reality of radiation-induced forces. A central element in deriving radiation pressure lies in the Maxwell stress tensor and the conservation laws of energy and momentum in electrodynamics, as presented in standard texts by Jackson [6], Griffiths [7], and Landau & Lifshitz [8].

Literature review. The transition from field theory to measurable force relies on the evaluation of electromagnetic momentum flux across surfaces. This formulation naturally yields the well-known expressions I/c for absorption and 2I/c for reflection, providing a consistent theoretical framework for understanding radiation forces in diverse settings. The study of light's momentum has also sparked one of the longest-standing debates in classical physics: the Abraham–Minkowski controversy regarding the correct form of electromagnetic momentum in media. Foundational contributions by Minkowski [9] and Abraham [10], and modern analyses such as Barnett [11], Milonni & Boyd [12], and Pfeifer et al. [13] illustrate the richness of this problem and its importance for both theoretical and applied optics. Beyond theoretical considerations, radiation pressure underpins numerous modern technologies. The demonstration of angular momentum transfer by Beth [14] and the Nobel-winning work on laser cooling and trapping by Ashkin [15], Ashkin et al. [16], and Chu [17] have enabled entirely new experimental frontiers. Applications now range from optical tweezers and nanoparticle trapping [18] to proposals for photon-driven solar sails in space exploration. Recent reviews, such as Dholakia & Zemánek [19] and Padgett [20], further highlight the importance of radiation forces across scales, from single atoms to macroscopic systems. In this work, we revisit the derivation of radiation pressure directly from Maxwell's equations, providing a step-by-step pedagogical pathway from the field equations to the stress tensor formulation and the resulting expressions for pressure. By drawing upon both classical foundations and contemporary perspectives, we aim to present a unified and accessible treatment that will serve as a resource for students and researchers seeking a clear physical understanding of this central phenomenon in electrodynamics.

Research Methodology. We work in SI units, in vacuum, with fields: $E = \vec{E}(\vec{r}, t)$ and $B = \vec{B}(\vec{r}, t)$. Maxwell's equations are:

$$\nabla \cdot \mathrm{E} = \frac{\rho}{\varepsilon_0}\ ,\ \nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t}\ ,\ \nabla \cdot \mathrm{B} = 0\ ,\ \nabla \times \mathrm{B} = \mu_0 \mathrm{J} + \mu_0 \varepsilon_0 \frac{\partial \mathrm{E}}{\partial t} \tag{1}$$

where E is electric field, B is magnetic field, J is current density, ρ is charge density, ∇ is Nabla operator, and ε_0 and μ_0 are the vacuum permittivity and permeability; the speed of light is

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{2}$$

The Lorentz force density on charges and currents is

$$\mathrm{f} = \rho \mathrm{E} + \mathrm{J} \times \mathrm{B} \tag{3}$$

where f is Lorentz force density.

The method consists of the following steps: 1) energy conservation (Poynting's theorem) is obtained from Maxwell's equations, which introduces the energy density and the Poynting vector; 2) ρ and J are eliminated from the Lorentz force density with the help of Maxwell's equations, which yields the momentum-balance law and the Maxwell stress tensor; 3) the divergence theorem expresses the net force on matter as a surface integral; 4) the normal component of the stress tensor is evaluated for a monochromatic plane wave to obtain the pressure on a surface; 5) the result is generalized to reflection and transmission.

Energy conservation (Poynting's theorem). We take the dot product of E with the Ampère–Maxwell law and of B with Faraday's law:

$$\mathrm{E} \cdot (\nabla \times \mathrm{B}) = \mu_0 \, \mathrm{E} \cdot \mathrm{J} + \mu_0 \varepsilon_0 \, \mathrm{E} \cdot \frac{\partial \mathrm{E}}{\partial t}\ \ ;\ \ \mathrm{B} \cdot (\nabla \times \mathrm{E}) = -\mathrm{B} \cdot \frac{\partial \mathrm{B}}{\partial t} \tag{4}$$

From (4) we easly obtain:

$$\nabla \cdot (\mathrm{E} \times \mathrm{B}) = -\mathrm{B} \cdot \frac{\partial \mathrm{B}}{\partial t} - \mu_0 \, \mathrm{E} \cdot \mathrm{J} - \mu_0 \varepsilon_0 \, \mathrm{E} \cdot \frac{\partial \mathrm{E}}{\partial t} \tag{5}$$

$$\frac{\partial}{\partial t}\left(\frac{\varepsilon_0 E^2}{2}+\frac{B^2}{2\mu_0}\right)+\nabla\cdot\left(\frac{1}{\mu_0}\,\mathrm{E}\times\mathrm{B}\right)=-\mathrm{E}\cdot\mathrm{J} \tag{6}$$

This is Poynting's theorem:

$$\frac{\partial u}{\partial t}+\nabla\cdot\mathrm{S}=-\mathrm{E}\cdot\mathrm{J} \tag{7}$$

with the electromagnetic energy density and the Poynting vector

$$u=\frac{\varepsilon_0 E^2}{2}+\frac{B^2}{2\mu_0},\qquad \mathrm{S}=\frac{1}{\mu_0}\,\mathrm{E}\times\mathrm{B} \tag{8}$$

In a charge-free, current-free region ($\rho=0$, $\mathrm{J}=0$) energy is locally conserved:

$$\frac{\partial u}{\partial t}+\nabla\cdot\mathrm{S}=0 \tag{9}$$

Momentum conservation: Field momentum and stress tensor. In the Lorentz force density (6) we substitute $\rho=\varepsilon_0\,\nabla\cdot\mathrm{E}$ and $\mathrm{J}=\frac{1}{\mu_0}\nabla\times\mathrm{B}-\varepsilon_0\,\partial_t\mathrm{E}$, both following from Maxwell's equations:

$$\mathrm{f}=\varepsilon_0(\nabla\cdot\mathrm{E})\,\mathrm{E}+\frac{1}{\mu_0}(\nabla\times\mathrm{B})\times\mathrm{B}-\varepsilon_0\,\frac{\partial\mathrm{E}}{\partial t}\times\mathrm{B} \tag{10}$$

To turn the last term into a time derivative we use Faraday's law:

$$\frac{\partial}{\partial t}(\mathrm{E}\times\mathrm{B})=\frac{\partial\mathrm{E}}{\partial t}\times\mathrm{B}+\mathrm{E}\times\frac{\partial\mathrm{B}}{\partial t}=\frac{\partial\mathrm{E}}{\partial t}\times\mathrm{B}-\mathrm{E}\times(\nabla\times\mathrm{E}) \tag{11}$$

Hence

$$\mathrm{f}=-\varepsilon_0\frac{\partial}{\partial t}(\mathrm{E}\times\mathrm{B})+\varepsilon_0[(\nabla\cdot\mathrm{E})\,\mathrm{E}-\mathrm{E}\times(\nabla\times\mathrm{E})]+\frac{1}{\mu_0}(\nabla\times\mathrm{B})\times\mathrm{B} \tag{12}$$

Now we apply identity (8) (noting that $-\mathrm{E}\times(\nabla\times\mathrm{E})=(\nabla\times\mathrm{E})\times\mathrm{E}$) with $\mathrm{A}=\mathrm{E}$ and $\mathrm{A}=\mathrm{B}$, and, for symmetry, add the term $\frac{1}{\mu_0}(\nabla\cdot\mathrm{B})\mathrm{B}$, which vanishes because $\nabla\cdot\mathrm{B}=0$. The result is

$$\mathrm{f}=-\frac{\partial\mathrm{g}}{\partial t}+\varepsilon_0\left[(\nabla\cdot\mathrm{E})\mathrm{E}+(\mathrm{E}\cdot\nabla)\mathrm{E}-\tfrac{1}{2}\nabla E^2\right]+\frac{1}{\mu_0}\left[(\nabla\cdot\mathrm{B})\mathrm{B}+(\mathrm{B}\cdot\nabla)\mathrm{B}-\tfrac{1}{2}\nabla B^2\right] \tag{13}$$

where the electromagnetic momentum density is

$$\mathrm{g}=\varepsilon_0\,\mathrm{E}\times\mathrm{B}=\frac{\mathrm{S}}{c^2} \tag{14}$$

The terms in square brackets can be written as a total divergence:

$$\varepsilon_0\;\partial_j\big(E_iE_j\big)=\varepsilon_0[(\nabla\cdot\mathrm{E})E_i+(\mathrm{E}\cdot\nabla)E_i]\ \text{and}\ \partial_i(1/2\,E^2)=\partial_j\big(1/2\,\delta_{ij}E^2\big) \tag{15}$$

(and likewise for the magnetic field). In this way we obtain the momentum-balance law and the symmetric Maxwell stress tensor σ:

$$\frac{\partial\mathrm{g}}{\partial t}-\nabla\cdot\sigma=-\mathrm{f},\qquad(\nabla\cdot\sigma)_i=\textstyle\sum_j\frac{\partial\sigma_{ij}}{\partial x_j} \tag{16}$$

$$\sigma_{ij}=\varepsilon_0\left(E_iE_j-\tfrac{1}{2}\delta_{ij}E^2\right)+\frac{1}{\mu_0}\left(B_iB_j-\tfrac{1}{2}\delta_{ij}B^2\right) \tag{17}$$

Integrating the balance law over a volume V and applying the divergence (Gauss–Ostrogradsky) theorem gives the net force on matter inside V:

$$\mathrm{F}=\textstyle\int_V\mathrm{f}\,dV=-\frac{d}{dt}\int_V\mathrm{g}\,dV+\oint_{\partial V}\sigma\cdot d\mathrm{A} \tag{18}$$

Thus, light exerts a mechanical force because the electromagnetic field carries momentum whose flux through a surface is transferred to matter. For time-averaged (steady) fields the first term vanishes and the force is determined solely by the flux of the stress tensor through the surface.

Pressure on a surface: general statement. Let $\hat{\mathbf{n}}$ be the outward unit normal of a surface element dA of the body. The traction (force per unit area) that the field exerts on the surface and the normal pressure are

$$\mathrm{t}=\sigma\cdot\hat{\mathrm{n}},\qquad p=-\hat{\mathrm{n}}\cdot\mathrm{t}=-\,\hat{\mathrm{n}}\cdot\sigma\cdot\hat{\mathrm{n}} \tag{19}$$

A positive pressure corresponds to compression of the surface (force directed inward). Now consider a monochromatic plane wave propagating along $+\hat{\mathrm{z}}$:

$$\mathrm{E}(z,t)=E_0\cos(kz-\omega t)\,\hat{\mathrm{x}},\qquad \mathrm{B}(z,t)=\frac{E_0}{c}\cos(kz-\omega t)\,\hat{\mathrm{y}} \tag{20}$$

For this wave (since $B=E/c$, so that $B^2/\mu_0=\varepsilon_0E^2$):

$$u=\frac{\varepsilon_0E^2}{2}+\frac{B^2}{2\mu_0}=\varepsilon_0E^2,\qquad \mathrm{S}=\frac{1}{\mu_0}\,\mathrm{E}\times\mathrm{B}=\varepsilon_0cE^2\,\hat{\mathrm{z}} \tag{21}$$

Time averages over many cycles give $\langle E^2\rangle=E_0^2/2$ and

$$\langle u\rangle=\varepsilon_0\frac{E_0^2}{2},\qquad \langle\mathrm{S}\rangle=\varepsilon_0c\,\frac{E_0^2}{2}\,\hat{\mathrm{z}}\equiv I\,\hat{\mathrm{z}} \tag{22}$$

where $I=\langle S\rangle$ is the intensity; note also that $I=c\langle u\rangle$. For the stress-tensor component normal to a surface $z=\mathrm{const}$ (the wave is transverse, so $E_z=B_z=0$) we have

$$\sigma_{zz} = \varepsilon_0\left(E_z^2 - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(B_z^2 - \frac{1}{2}B^2\right) = -\frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = -u \quad (23)$$

Perfectly absorbing surface and perfect mirror. For a perfectly absorbing surface (no reflected or transmitted wave in front of the surface) the outward normal is $\hat{\mathrm{n}} = -\hat{\mathrm{z}}$, so by (24) and (28) the time-averaged normal pressure is

$$\langle p\rangle = -\langle\sigma_{zz}\rangle = \langle u\rangle = \frac{I}{c} \quad (24)$$

For a perfectly reflecting mirror at normal incidence the momentum carried by the wave reverses upon reflection: the incident momentum flux I/c is delivered to the mirror, and the reflected wave carries away an equal momentum flux I/c in the $-\hat{\mathrm{z}}$ direction. The momentum transfer is therefore doubled:

$$\langle p\rangle_{\mathrm{mirror}} = \frac{I}{c} + \frac{I}{c} = \frac{2I}{c} \quad (25)$$

General normal-incidence balance with reflection and transmission. Let I_{inc}, I_{refl} and I_{trans} be the cycle-averaged incident, reflected and transmitted intensities. The momentum flux normal to the surface equals I/c. Conservation of momentum across a pillbox yields the pressure on the interface (on the front surface):

$$p = \frac{I_{\mathrm{inc}} + I_{\mathrm{refl}} - I_{\mathrm{trans}}}{c} = \frac{(1+R-T)\,I_{\mathrm{inc}}}{c} \quad (26)$$

where $R = I_{\mathrm{refl}}/I_{\mathrm{inc}}$ and $T = I_{\mathrm{trans}}/I_{\mathrm{inc}}$. Special cases: an absorbing wall ($R = T = 0$) gives $p = I/c$; a perfect mirror ($R = 1, T = 0$) gives $p = 2I/c$; a lossless single interface ($R + T = 1$) gives $p = 2R\,I_{\mathrm{inc}}/c$.

The field momentum density $\mathrm{g} = \mathrm{S}/c^2$ shows that wherever there is energy flow, there is momentum flow. The Maxwell stress tensor σ is the momentum-flux density of the field: its normal component gives the pressure, and its off-diagonal components encode shear forces. The pressure equals the change in normal momentum flux delivered by the fields to matter.

As an example, sunlight at the Earth's orbit has intensity $I \approx 1360\ \mathrm{W}\cdot\mathrm{m}^{-2}$. Then

$$p_{\mathrm{abs}} = \frac{I}{c} \approx \frac{1360\ \mathrm{W\cdot m^{-2}}}{3\cdot 10^8\ \mathrm{m/s}} \approx 4.5\cdot 10^{-6}\ \mathrm{Pa}, \qquad p_{\mathrm{mirror}} \approx 9.1\cdot 10^{-6}\ \mathrm{Pa} \quad (27)$$

Conclusion. Maxwell's equations imply that electromagnetic fields carry momentum; the associated momentum-balance law introduces the Maxwell stress tensor. The flux of this tensor through a surface produces a mechanical force. For electromagnetic waves this manifests as a radiation pressure equal to I/c for absorption and $2I/c$ for reflection at normal incidence. Thus, the pressure of light is a direct, quantitative consequence of classical electrodynamics. For sunlight at the Earth's orbit it amounts to about $4.5\cdot 10^{-6}$ Pa on a perfect absorber and $9.1\cdot 10^{-6}$ Pa on a perfect mirror.

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ОБРАТНАЯ ЗАДАЧА ДЛЯ ОДНОМЕРНОГО ГИПЕРБОЛИЧЕСКОГО УРАВНЕНИЯ С ПОСТОЯННЫМ ЗАПАЗДЫВАЮЩИМ АРГУМЕНТОМ

Аннотация. *В данной статье рассматривается обратная задача для одномерного линейного гиперболического уравнения с постоянными коэффициентами и постоянным запаздывающим аргументом – задача определения неизвестной функции источника* $f(t)$. *В отличие от прямой задачи, здесь в качестве дополнительной информации задается значение решения во внутренней точке области. Постановка обратной задачи, существование и единственность ее решения обоснованы полностью доказанной теоремой. Решение строится с использованием метода Фурье и аппарата интегральных уравнений Вольтерра. Полученные результаты могут быть использованы при идентификации и управлении динамическими системами с запаздыванием.*

Ключевые слова: *обратная задача, гиперболическое уравнение, запаздывающий аргумент, функция источника, метод Фурье, существование и единственность.*

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O'ZGARMAS KECHIKUVCHI ARGUMENTGA EGA BIR O'LCHOVLI GIPERBOLIK TENGLAMA UCHUN TESKARI MASALA

Annotatsiya. *Ushbu maqolada o'zgarmas koeffitsiyentli va o'zgarmas kechikuvchi argumentga ega bir o'lchovli chiziqli giperbolik tenglama uchun teskari masala – noma'lum manba funksiyasini aniqlash masalasi ko'rib chiqiladi. To'g'ri masaladan farqli ravishda, bu yerda qo'shimcha ma'lumot sifatida sohaning ichki nuqtasidagi yechimning qiymati beriladi. Teskari masalaning qo'yilishi, uning yechimi mavjudligi va yagonaligi to'liq isbotlangan teorema orqali asoslanadi. Masalani yechish uchun Furye usuli va Volterra integral tenglamalari apparatidan foydalaniladi. Olingan natijalardan kechikishga ega dinamik tizimlarni identifikatsiyalash va boshqarishda foydalanish mumkin.*

Kalit so'zlar: *teskari masala, giperbolik tenglama, kechikuvchi argument, manba funksiyasi, Furye usuli, mavjudlik va yagonalik.*

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INVERSE PROBLEM FOR A ONE-DIMENSIONAL HYPERBOLIC EQUATION WITH A CONSTANT DELAYED ARGUMENT

Annotation. *This paper considers an inverse problem for a one-dimensional linear hyperbolic equation with constant coefficients and a constant delayed argument, namely, the problem of determining*