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KONSOL STERJENNING EGILISHGA BIKIRLIGINI ERKIN TEBRANISHLAR VA NAZARIY MODEL ASOSIDA ANIQLASH

Annotatsiya. *Ushbu maqolada bir uchi qistirib mahkamlangan, ikkinchi uchi erkin (konsol) sterjenning egilishga bikirligini ikkita usul yordamida aniqlash masalasi ko‘rib chiqilgan. Sterjenning erkin so‘nuvchi ko‘ndalang tebranish jarayonini eksperimental qayd etish va uning grafigidan foydalanib chastota tavsiflarini ajratib olish orqali Eyler–Bernulli dinamik tenglamasi va konsol balkaning chegaraviy shartlaridan kelib chiqadigan xarakteristik (chastota) tenglama asosida tebranish davri, chastotasi, logarifmik dekrement hamda so‘nish koeffitsiyenti hisoblangan va EIning sonli qiymati topilgan. Olingan natijalar asosida taqqoslash ishlari bajarilgan.*

Kalit so‘zlar: *Eyler-Bernulli tenglamasi, konsol balka, chastota, egilish bikirligi (EI), logarifmik dekrement.*

ОПРЕДЕЛЕНИЕ ИЗГИБНОЙ ЖЁСТКОСТИ КОНСОЛЬНОГО СТЕРЖНЯ НА ОСНОВЕ СВОБОДНЫХ КОЛЕБАНИЙ И ТЕОРЕТИЧЕСКОЙ МОДЕЛИ

Аннотация. *В данной статье рассматривается задача определения изгибной жёсткости стержня, один конец которого жёстко защемлён, а другой свободен (консольного стержня), двумя методами. Путём экспериментальной регистрации процесса свободных затухающих поперечных колебаний стержня и извлечения частотных характеристик по его графику, на основе динамического уравнения Эйлера–Бернулли и характеристического (частотного) уравнения, вытекающего из граничных условий консольной балки, были рассчитаны период, частота колебаний, логарифмический декремент и коэффициент затухания, а также найдено численное значение EI. На основе полученных результатов выполнены сравнительные работы.*

Ключевые слова: *уравнение Эйлера–Бернулли, консольная балка, частота, изгибающий момент (EI), логарифмический декремент.*

DETERMINATION OF THE FLEXURAL RIGIDITY OF A CANTILEVER BEAM BASED ON FREE VIBRATIONS AND A THEORETICAL MODEL

Annotation. *This article examines the problem of determining the flexural rigidity of a rod–specifically a cantilever rod fixed at one end and free at the other–using two methods. By experimentally recording the process of free damped transverse oscillations and extracting frequency characteristics from the resulting plot, the oscillation period and frequency, logarithmic decrement, and damping coefficient were calculated, and the numerical value of flexural rigidity (EI) was determined; these calculations were based on the dynamic Euler-Bernoulli equation and the characteristic (frequency) equation derived from the cantilever beam's boundary conditions. A comparative analysis was conducted based on the obtained results.*

Keywords: *Euler–Bernoulli equation, cantilever beam, frequency, bending stiffness (EI), logarithmic decrement.*

Kirish. Konstruksiya elementlarining egilishga bikirligi *EI* sterjen, balka va boshqa elementlarning statik va dinamik harakatlarini baholashda asosiy mexanik xarakteristikalardan biri hisoblanadi [1]. *EI* ni aniqlashning ikki asosiy yo‘li mavjud: statik usul (ma’lum yuklama ostida egilishni o‘lchash) va dinamik usul (erkin yoki majburiy tebranish chastotasidan foydalanish) [2]. Dinamik usulning afzalligi shundaki, bunda sterjenning bikirligini tez va bir necha marta qayta aniqlash imkonini beradi [3].

Ushbu ishda vertikal joylashtirilgan pastki uchi qistirib mahkamlangan sterjenning so‘nuvchi tebranish grafigidan foydalanib uning egilish bikirligi *EI* ni aniqlash, topilgan qiymatlarni Eyler-Bernulli dinamik tenglamasidan olingan natijalarga muvofiq tekshirish va natijalarni tarjiba bilan taqqoslashdan iborat.

Masalaning qo‘yilishi. Eyler-Bernulli gipotezasi asosidagi kichik egilishlar nazariyasida balka o‘qining ko‘ndalang yuk *q(x)* ta’siridagi egilish chizig‘i *y(x)* quyidagi to‘rtinchi tartibli differensial tenglama bilan tavsiflanadi:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}=0 \quad (1)$$

Bu yerda: ρ-material zichligi, A-ko‘ndalang kesim yuzi.

(1) tenglamani yechimni o‘zgaruvchilarni ajratish usulida $y(x,t)=y(x)\cdot T(t)$ ko‘rinishida izlaymiz natijada tenglama quyidagi ko‘rinishidagi

$$EIy^{IV}(x)\cdot T(t)+\rho\cdot A\cdot y(x)\cdot T''(t)=0 \quad (2)$$

differensial tenglamaga ega bo‘lamiz.

Bu yerda: $y(x,t)$ -x nuqtadagi ko‘ndalang siljish, ρA -birlik uzunlikdagi massa.

(2)tenglamaning ikkala tomonini $y(x)$ ga bo‘lib belgilash orqali quyidagi tenglikka ega bo‘lamiz:

$$y^{IV}-\beta^4\cdot y=0 \quad (3)$$

Bunda

$$-\frac{\rho\cdot A\cdot T''(t)}{EI\cdot T(t)}=\beta^4$$

U holda (3) differensil tenglamani yechimi

$$y(x)=C_1e^{\beta x}+C_2e^{-\beta x}+C_3e^{i\beta x}+C_4e^{-i\beta x} \quad (4)$$

ko‘rinishni oladi. Olingan (4) yechimni Eyler fo‘rmulalari yordamida uni haqiqiy funksiya ko‘rinishiga o‘tkazamiz.

$$e^{\beta x}+e^{-\beta x}=\cosh(\beta x)+\sinh(\beta x) \quad (a)$$

$$e^{i\beta x}+e^{-i\beta x}=\cos(\beta x)+\sin(\beta x) \quad (b)$$

$$y(x)=C_1\cosh(\beta x)+C_2\sinh(\beta x)+C_3\cos(\beta x)+C_4\sin(\beta x) \quad (5)$$

(5) funksiyaning hosilalarini topamiz.

$$y'(x)=C_1\beta sinh(\beta x)+C_2\beta cosh(\beta x)-C_3\beta sin(\beta x)+C_4\beta cos(\beta x)$$

$$y''(x)=C_1\beta^2\cosh(\beta x)+C_2\beta^2\sinh(\beta x)-C_3\beta^2cos(\beta x)-C_4\beta^2sin(\beta x)$$

$$y'''(x)=C_1\beta^3\sinh(\beta x)+C_2\beta^3\cosh(\beta x)+C_3\beta^3sin(\beta x)-C_4\beta^3cos(\beta x)$$

Boshlang‘ich shartlardan

$$x=0 \quad da \quad y(0)=0 \qquad C_1+C_3=0$$

$$x=0 \quad da \quad y'(0)=0 \qquad C_2+C_4=0$$

bundan ko‘rinadiki, sterjenning mahkamlangan uchida egilish bo‘lmasligini bildiradi.

$$x=L \quad da \quad y''(0)=0 \quad C_1\cosh(\beta L)+C_2\sinh(\beta L)-C_3cos(\beta L)-C_4sin(\beta L)=0$$

$$x=L \quad da \quad y'''(0)=0 \quad C_1\sinh(\beta L)+C_2\cosh(\beta L)+C_3sin(\beta L)-C_4cos(\beta L)=0$$

$$C_1=-C_3 \quad C_2=-C_4 \quad C_2=-C_1\frac{\cosh(\beta L)+\cos(\beta L)}{\sinh(\beta L)+\sin(\beta L)} \quad (6)$$

$C_1 = C_2 = 0$ dan boshqa yechim mavjud bo‘lishi uchun sistemaning determinanti nolga teng bo‘lishi shart:

$$\Delta=\begin{vmatrix}cosh(\beta L)+cos(\beta L) & sinh(\beta L)+sin(\beta L\\ sinh(\beta L)-sin(\beta L) & cosh(\beta L)+cos(\beta L)\end{vmatrix}=0 \quad (7)$$

Determinantni hisoblasak, u holda quyidagi ko‘rinishni oladi:

$$1+2cosh(\beta L)cos(\beta L)+1=0 \quad (8)$$

Natijada chastota xarakteristik tenglamasi hosil bo‘ladi:

$$cosh(\beta L)\cdot cos(\beta L)=-1.$$

Xos qiymatlar β_nLning ildizlari quyidagi jadvaldan aniqlanadi.

n	β_nL
1	1,8751
2	4,6941
3	7,8548
4	10,9955
n ≥ 5	≈ (2n−1)·π/2

Yechish usuli. Endi vaqt bo‘yicha olingan differensila tenglamani yechamiz:

$$\frac{\rho \cdot A \cdot T''}{EI} + \beta^4 T = 0; \qquad T = e^{\lambda t} \tag{9}$$

U holda (9) tenglamani yechimi quyidagicha bo‘ladi:

$$T(t) = Acos(\omega t) + Bsin(\omega t) \tag{10}$$

Bu yerda ω - sterjenning burchak chastotasi.

Olingan (4) va (10) yechimlar orqali (1) tenglamaning umumiy yechimi quyidagi

$$y(x) = [C_1\cosh(\beta x) + C_2\sinh(\beta x) + C_3\cos(\beta x) + C_4\sin(\beta x)] \cdot [Acos(\omega t) + Bsin(\omega t)] \tag{11}$$

ko‘rinishni oladi. Demak, tebranishning chastotasi ω_n, sterjenning birlik uzunlikka to‘g‘ri keladigan massasi ρA hamda uzunligi L ma’lum bo‘lsa, EI ni bevosita hisoblash mumkin.

Chegaraviy shartlarni qo‘llash natijasida olingan yakuniy chastota tenglamasi:

$$cosh(\beta L) \cdot cos(\beta L) = -1.$$

Bu yerdagi determinant sistemaning bir jinsli tenglamalari uchun quyidagicha yoziladi:

$$\Delta(\beta L) = 2[1 + cosh(\beta L) \cdot cos(\beta L)] = 0 \tag{12}$$

Endi EI egilish bikrligini topish uchun (9) tenlamani λ belgilashda olingan bog‘lanishdan foydalanamiz:

$$\frac{\rho \cdot A}{EI} \cdot \lambda^2 + \beta^4 = 0 \qquad \lambda^2 = -\beta \frac{EI}{\rho A} \tag{13}$$

Natijada yechim quyidagi ko‘rinishda bo‘ladi

$$\lambda = \pm i\beta^2 \sqrt{\frac{EI}{\rho A}} = \pm i\omega; \quad \omega_2 = \frac{\beta^4 EI}{\rho A} \tag{14}$$

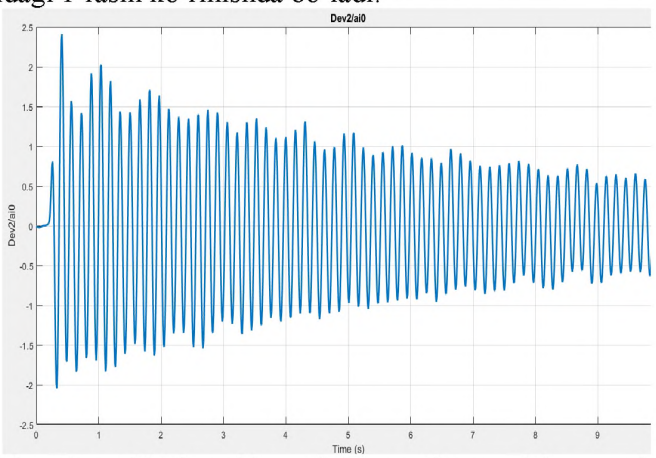
$$EI = \frac{\rho A \omega_2}{\beta^4} \quad \text{yoki} \quad EI = \frac{{\omega_n}^2 \cdot \rho A \cdot L^4}{(\beta_1 L)^4} \tag{15}$$

O‘tkazilgan tajribalar ularni tahlil qilish. Tekshirilayotgan sterjen vertikal holatda o‘rnatilgan, pastki uchi qistirib mahkamlangan, yuqori uchi esa erkin. Sterjen uchun asosiy parametrlarini quyidagicha kiritamiz:

Uzunligi: $L = 80\ sm$; To‘liq massasi: $m = 800\ g$; birlik uzunlikka to‘g‘ri keladigan massa: $\rho A = 1{,}000\ kg/m$.

Sterjenning yuqori uchi qo‘lda kichik boshlang‘ich siljish bilan tebranishga keltirilgan va so‘ngra erkin qo‘yib yuborilgan. Erkin tebranish jarayoni analog signal (kuchlanish, V) ko‘rinishida sensor va ma’lumot yig‘ish tizimi (LabVIEW, fayl: s-Messung_077.lvm) yordamida qayd etilgan. Namuna olish chastotasi 100 Hz (Δt = 0,01 s) bo‘lib, jami 1000 ta nuqta (10 sekundlik yozuv) olingan.

Tebranish grafigi quyidagi 1-rasm ko‘rinishda bo‘ladi:



1-rasm. Sterjenning erkin so‘nuvchi tebranish grafigi (nuqtalar – aniqlangan cho‘qqilar, uzuq chiziq – amplituda konverti).

Grafikdan ko‘rinib turibdiki, boshlang‘ich impulsdan (t ≈ 0,2–0,4 s) so‘ng sterjen deyarli bir xil davr bilan, ammo asta-sekin kamayuvchi amplitudada tebranmoqda. Bu klassik so‘nuvchi tebranish xarakteriga mos keladi. Signaldan avtomatik ravishda aniqlangan jami 61 ta cho‘qqidan har beshinchisi quyida keltirilgan (to‘liq jadval boshlang‘ich s-Messung_077.lvm faylida mavjud):

№	t(s)	Amplituda
1	0.560	1.5714
6	1.340	1.4379
11	2.120	1.4677
16	2.900	1.4226

21	3.680	1.2341
26	4.460	1.0648
31	5.240	0.9831
36	6.020	0.9178
41	6.800	0.9092
46	7.580	0.7864
51	8.360	0.6285
56	9.130	0.6230
61	9.910	0.5818

61 ta ketma-ket cho‘qqi orasidagi vaqt farqlarining o‘rtachasi sifatida tebranish davri topildi: $T = 0{,}1558\ s$ (standart chetlanish ±0,005 s, bu 0,01 s namuna olish qadamining kvantlash xatosiga mos keladi). Bundan so‘nuvchi (damped) chastota:

$$f_T = \frac{1}{T} = \frac{1}{0{,}1558} \approx 6{,}417\ Hz \qquad \omega_d = \frac{2\pi}{T_d} \approx 40{,}32 \text{ rad/s}.$$

Logarifmik dekrement ikki mustaqil usulda baholandi:

Klassik usul (birinchi va oxirgi cho‘qqi, n = 60 davr):

$$\delta_1 = \left(\frac{1}{60}\right) \cdot ln\left(\frac{x_1}{x_{61}}\right) = \left(\frac{1}{60}\right) \cdot ln\left(\frac{1{,}5714}{0{,}5818}\right) \approx 0{,}0176;$$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = \frac{0{,}0176}{\sqrt{(39{,}48 + 0{,}0003)}} \approx 0{,}0028 \text{ (ya’ni} \approx 0{,}28\ \%).$$

Bu qiymat $\zeta \ll 1$ shartini to‘liq qanoatlantiradi, demak, sterjenda so‘nish juda kuchsiz (yengil dempfirlangan tizim).

Xos chastotani aniqlash:

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}}) = \frac{40{,}32}{\sqrt{(1 - 0{,}0000078)}} = 40.32 \text{ rad/s}.$$

Kutilganidek, ζ juda kichik bo‘lgani uchun so‘nishsiz chastota amalda so‘nuvchi chastotadan farq qilmadi (farq 0,001%dan kam). (15) formulaga L = 0,80 m, ρA = 1,000 kg/m, $\beta_1 L$ = 1,8751 va yuqorida topilgan ω_n = 40,32 rad/s qiymatlarini qo‘yamiz;

$$EI = \frac{{\omega_n}^2 \cdot \rho A \cdot L^4}{(\beta_1 L)^4} = \frac{(40{,}32)^2 \cdot 1{,}000 \cdot (0{,}80)^4}{(1{,}8751)^4} = 53.86\ N \cdot m^2.$$

Endi bu topilgan *EI* ni taqqoslash uchun xuddi shu sterjen gorizontal holatda, bir uchi qistirib mahkamlangan konsol sxemada, ikkinchi erkin uchiga ketma-ket beshta turli yuk P qo‘yilib, mos egilish miqdori f_st o‘lchangan. Konsol balka uchun statik egilish formulasi $f_{st} = \frac{PL^3}{(3EI)}$ dan foydalanib, har bir yuklama uchun EI hisoblanadi:

1-jadval

Statik yuklama tajribasi natijalari (to‘ldiriladi)

№	Yuk $P(N)$	Egilish f (mm)	Sterjen uzunligi (m)	$EI = \frac{PL^3}{(3f)}$
1	2	1.06	0.44	53.57
2	4	2.1	0.44	54.08
3	6	3.01	0.44	56.6
4	8	4.11	0.44	55.21
O‘rtacha *EI*				54.865

To‘rtta o‘lchov bo‘yicha EI ning o‘rtacha qiymati va standart chetlanishi hisoblanadi: EI_{stat} $(\frac{1}{4})\Sigma$ EI_i

Natijalarni taqqoslash. Uchta yondashuv -(1) tebranish grafigidan bevosita, (1) Eyler–Bernulli dinamik tenglamasi asosida va tajribasi asosida topilgan *EI* qiymatlari quyidagi jadvalda solishtirilgan:

2-jadval

Turli usullar bo‘yicha aniqlangan EI qiymatlarini taqqoslash

Usul	EI N · m²	Izoh
1. Dinamik (erkin so‘nuvchi tebranish, grafik) usuli	53,86	Ushbu maqolada hisoblangan
2. Statik yuklama usuli (4 xil yuk, egilish)	54.865	Talaba tomonidan to‘ldiriladi
Nisbiy farq	1.87 %	$\Delta = \lvert EI_1 - EI_2 \rvert / EI_1 \times 100\%$

Ikkala usulda topilagn natijalar bir-biriga deyarli yaqin, chunki ular bir xil fizik sterjenning bikirligini tavsiflaydi.

Xulosa. Vertikal konsol sterjenning so‘nuvchi tebranish grafigidan foydalanib davr, chastotasi , logarifmik dekrement va so‘nish koeffitsiyenti aniqlandi. Eyler-Bernulli dinamik tenglamasi va konsol balkaning chegaraviy shartlaridan kelib chiqadigan xarakteristik tenglama asosida sterjenning egilish bikirligi topildi. Bu olingan natijalar taqqoslandi; ikkala usul natijalarining yaqinligi Eyler–Bernulli balka modelining ushbu sterjen uchun amaliy jihatdan asosli ekanligini tasdindi.

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SIMMETRIK MATRITSAVIY LAPLAS ALMASHTIRISHINING KOMPLEKS DINAMIK SISTEMALARGA TATBIQI

Annotatsiya. *Ushbu maqolada* 2×2 *kompleks simmetrik matritsalar fazosidagi* $F(Z) = Z - Z^2$ *akslantirish uchun invariant tortuvchi sohada Abel tenglamasining golomorf yechimi topilib, nol nuqtada Frechet differensiali birlik operatorga tengligi va bu nuqta yakkalangan qo‘zg‘almas nuqta emasligi ko‘rsatilgan. Shuningdek,* $W = Z^{-1}$ *almashtirish va bosh matritsaviy logarifm yordamida taqribiy koordinata aniqlangan. Qoldiqning kvadratik kamayish bahosi asosida bartaraf etuvchi qatorning kompaktlarda tekis yaqinlashishi isbotlanib, yechimning asimptotik formulasi olingan hamda qoldiqning simmetrik matritsaviy Laplas tasviriga ega bo‘lishi shartidan Fatu koordinatasining integral ko‘rinishi topilgan.*

Kalit so‘zlar: *simmetrik matritsaviy Laplas almashtirishi, Abel funksional tenglamasi, Fatu koordinatasi, Frechet differensial, qoldiq funksiya, invariant tortuvchi soha.*

ПРИМЕНЕНИЕ МАТРИЧНОГО ПРЕОБРАЗОВАНИЯ ЛАПЛАСА НА ОБЛАСТИ СИММЕТРИЧЕСКИХ МАТРИЦ К КОМПЛЕКСНЫМ ДИНАМИЧЕСКИМ СИСТЕМАМ

Аннотация. *В статье построено голоморфное решение уравнения Абеля в инвариантной притягивающей области для отображения* $F(Z) = Z - Z^2$ *в пространстве комплексных симметричных матриц размера* 2×2*. Показано, что дифференциал Фреше этого отображения в нулевой точке равен тождественному оператору, причём данная точка не является изолированной неподвижной точкой. С помощью замены* $W = Z^{-1}$ *и главного матричного логарифма определена приближённая координата. На основе оценки квадратичного убывания остатка доказана равномерная сходимость на компактах ряда, устраняющего остаток, и получена асимптотическая формула решения. При условии, что остаток допускает представление в виде симметричного матричного преобразования Лапласа, получено интегральное представление координаты Фату.*

Ключевые слова: *матричное преобразование Лапласа на пространстве симметрических матриц, функциональное уравнение Абеля, координата Фату, дифференциал Фреше, остаточная функция, инвариантная область притяжения.*

APPLICATION OF THE SYMMETRIC MATRIX LAPLACE TRANSFORM TO COMPLEX DYNAMICAL SYSTEMS

Annotation. *This paper constructs a holomorphic solution of the Abel equation on an invariant attracting domain for the mapping* $F(Z) = Z - Z^2$ *in the space of* 2×2 *complex symmetric matrices. The Fréchet derivative of this mapping at the origin is shown to be the identity operator, while the origin is not an isolated fixed point. An approximate coordinate is defined using the substitution* $W = Z^{-1}$ *and the principal matrix logarithm. A quadratic decay estimate for the residual establishes uniform convergence on compact subsets of the series that cancels the residual, yielding an asymptotic formula for the solution. Under the assumption that the residual admits a symmetric matrix Laplace transform representation, an integral representation of the Fatou coordinate is obtained.*

Keywords: *symmetric matrix Laplace transform, Abel functional equation, Fatou coordinate, Fréchet differential, remainder function, invariant attracting domain.*

Kirish. Laplas almashtirishining matritsa argumentli ko‘rinishi bir nechta mustaqil o‘zgaruvchini yagona algebraik obyekt orqali ifodalash imkonini beradi. Simmetrik matritsalar fazosida bu yondashuv