Xulosa. Vertikal konsol sterjenning so‘nuvchi tebranish grafigidan foydalanib davr, chastotasi , logarifmik dekrement va so‘nish koeffitsiyenti aniqlandi. Eyler-Bernulli dinamik tenglamasi va konsol balkaning chegaraviy shartlaridan kelib chiqadigan xarakteristik tenglama asosida sterjenning egilish bikirligi topildi. Bu olingan natijalar taqqoslandi; ikkala usul natijalarining yaqinligi Eyler–Bernulli balka modelining ushbu sterjen uchun amaliy jihatdan asosli ekanligini tasdindi.

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SIMMETRIK MATRITSAVIY LAPLAS ALMASHTIRISHINING KOMPLEKS DINAMIK SISTEMALARGA TATBIQI

Annotatsiya. *Ushbu maqolada* 2×2 *kompleks simmetrik matritsalar fazosidagi* $F(Z) = Z - Z^2$ *akslantirish uchun invariant tortuvchi sohada Abel tenglamasining golomorf yechimi topilib, nol nuqtada Frechet differensiali birlik operatorga tengligi va bu nuqta yakkalangan qo‘zg‘almas nuqta emasligi ko‘rsatilgan. Shuningdek,* $W = Z^{-1}$ *almashtirish va bosh matritsaviy logarifm yordamida taqribiy koordinata aniqlangan. Qoldiqning kvadratik kamayish bahosi asosida bartaraf etuvchi qatorning kompaktlarda tekis yaqinlashishi isbotlanib, yechimning asimptotik formulasi olingan hamda qoldiqning simmetrik matritsaviy Laplas tasviriga ega bo‘lishi shartidan Fatu koordinatasining integral ko‘rinishi topilgan.*

Kalit so‘zlar: *simmetrik matritsaviy Laplas almashtirishi, Abel funksional tenglamasi, Fatu koordinatasi, Frechet differensial, qoldiq funksiya, invariant tortuvchi soha.*

ПРИМЕНЕНИЕ МАТРИЧНОГО ПРЕОБРАЗОВАНИЯ ЛАПЛАСА НА ОБЛАСТИ СИММЕТРИЧЕСКИХ МАТРИЦ К КОМПЛЕКСНЫМ ДИНАМИЧЕСКИМ СИСТЕМАМ

Аннотация. *В статье построено голоморфное решение уравнения Абеля в инвариантной притягивающей области для отображения* $F(Z) = Z - Z^2$ *в пространстве комплексных симметричных матриц размера* 2×2*. Показано, что дифференциал Фреше этого отображения в нулевой точке равен тождественному оператору, причём данная точка не является изолированной неподвижной точкой. С помощью замены* $W = Z^{-1}$ *и главного матричного логарифма определена приближённая координата. На основе оценки квадратичного убывания остатка доказана равномерная сходимость на компактах ряда, устраняющего остаток, и получена асимптотическая формула решения. При условии, что остаток допускает представление в виде симметричного матричного преобразования Лапласа, получено интегральное представление координаты Фату.*

Ключевые слова: *матричное преобразование Лапласа на пространстве симметрических матриц, функциональное уравнение Абеля, координата Фату, дифференциал Фреше, остаточная функция, инвариантная область притяжения.*

APPLICATION OF THE SYMMETRIC MATRIX LAPLACE TRANSFORM TO COMPLEX DYNAMICAL SYSTEMS

Annotation. *This paper constructs a holomorphic solution of the Abel equation on an invariant attracting domain for the mapping* $F(Z) = Z - Z^2$ *in the space of* 2×2 *complex symmetric matrices. The Fréchet derivative of this mapping at the origin is shown to be the identity operator, while the origin is not an isolated fixed point. An approximate coordinate is defined using the substitution* $W = Z^{-1}$ *and the principal matrix logarithm. A quadratic decay estimate for the residual establishes uniform convergence on compact subsets of the series that cancels the residual, yielding an asymptotic formula for the solution. Under the assumption that the residual admits a symmetric matrix Laplace transform representation, an integral representation of the Fatou coordinate is obtained.*

Keywords: *symmetric matrix Laplace transform, Abel functional equation, Fatou coordinate, Fréchet differential, remainder function, invariant attracting domain.*

Kirish. Laplas almashtirishining matritsa argumentli ko‘rinishi bir nechta mustaqil o‘zgaruvchini yagona algebraik obyekt orqali ifodalash imkonini beradi. Simmetrik matritsalar fazosida bu yondashuv

matritsa iziga asoslangan skalyar yadro va matritsa qiymatli original o‘rtasidagi munosabatni o‘rganishga olib keladi [1;6-b.].

Kompleks dinamikada multiplikatori birga teng bo‘lgan bir o‘zgaruvchili akslantirishning iteratsiyalari Fatu koordinatasi yordamida siljish dinamikasiga keltiriladi. Agar $f(0) = 0$ va $f'(0) = 1$ bo‘lsa, tegishli tortuvchi sohada golomorf ϕ funksiya

$$\phi\big(f(z)\big) = \phi(z) + 1 \tag{1}$$

Abel funksional tenglamasini qanoatlantiradi. Mazkur tenglama iteratsion jarayonni $\zeta \mapsto \zeta + 1$ parallel ko‘chirishga aylantiradi. Ushbu g‘oya matritsa argumentli akslantirishga o‘tkazilganda skalyar birlik o‘rnini $I^{(2)}$ birlik matritsa egallaydi.

Ushbu ishda S_2^* kompleks simmetrik 2×2 matritsalar fazosidagi quyidagi

$$F(Z) = Z - Z^2 \tag{2}$$

akslantirishni qaraymiz. Maqsad $W = Z^{-1}$ koordinatada qoldiqni bartaraf etuvchi normal yaqinlashuvchi qatorni qurish, undan matritsaviy Abel tenglamasining golomorf yechimini olish va qoldiqning simmetrik matritsaviy Laplas tasviri mavjud bo‘lganda, shu yechimni integral shaklda yozishdan iborat.

Mavzuga oid adabiyotlarning tahlili. C.S.Herz matritsa argumentli Bessel funksiyalarini [2], R.M.Joshi va J.M.C.Joshi umumlashgan Laplas almashtirishlarini [3], A.M.Mathai va G.Pederzoli matritsaviy Laplas almashtirishlari va Uitteker funksiyalarini o‘rgangan [5]. So‘nggi tadqiqot ishlarida simmetrik matritsaviy Laplas almashtirishining xossalari, o‘rama va Dyumel formulalari analoglari olingan [1, 4]. Leau va Fatu parabolik dinamika asoslarini yaratgan [7–9], J.Ekal va S.M.Voronin sektorial Fatu koordinatalari va analitik tasnifni rivojlantirgan [10,11]. M.Hakim chiziqli qismi birlik operatorli ko‘p o‘lchovli akslantirishlarni [12], M.Abate rezidual indeks orqali parabolik egri chiziqlar mavjudligini [14], F.Brakki va L.Molino kvaziparabolik dinamikani o‘rgangan [15]. A.Dudko va D.Sauzin formal Fatu koordinatalarining analitik davom ettirilishi va Borel–Laplas yig‘indilanishini asoslagan [13]. Ushbu ishda esa qoldiqni ifodalash uchun simmetrik matritsaviy Laplas tasviridan foydalaniladi.

Simmetrik matritsalar va Laplas almashtirishi. Haqiqiy va kompleks simmetrik matritsalar fazolarini mos ravishda $S_m = \{A \in \mathbb{R}^{m\times m}: A^{\mathrm{T}} = A\}$ va $S_m^* = \{Z \in \mathbb{C}^{m\times m}: Z^{\mathrm{T}} = Z\}$ orqali belgilaymiz. S_m^* yozuvidagi yulduzcha qo‘shma transponirlash amalini emas, kompleks simmetrik matritsalar sinfining belgisini bildiradi. Maqola davomida $\mathrm{Sp}(\cdot) = \mathrm{tr}(\cdot)$ belgisi matritsaning izini, ya’ni asosiy diagonaldagi elementlar yig‘indisi anglatadi [6].

Har bir i, j uchun

$$\eta_{ij} = \begin{cases} 1, & i = j, \\ \frac{1}{2}, & i \neq j \end{cases} \tag{3}$$

deb olamiz va kompleks parametrni $Z = \big(\eta_{ij} z_{ij}\big)$ ko‘rinishda yozamiz. $A = \big(a_{ij}\big) \in S_m$ bo‘lsa, u holda $\mathrm{Sp}\big(ZA^{\mathrm{T}}\big) = \sum_{i,j=1}^{m} \eta_{ij}\, z_{ij} a_{ij} = \sum_{1\le i\le j\le m} z_{ij}\, a_{ij}$. Bunday shalkdan foydalanish diagonaldan tashqari elementlarning matritsa izi $\mathrm{Sp}\big(ZA^{\mathrm{T}}\big)$ yoyilmasida ikki marta hisoblanishini bartaraf etadi.

Misol uchun, $m = 2$ bo‘lganda

$$Z = \begin{pmatrix} z_{11} & z_{12}/2 \\ z_{12}/2 & z_{22} \end{pmatrix}, \qquad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$$

va

$$\mathrm{Sp}\big(ZA^{\mathrm{T}}\big) = \mathrm{Sp}(ZA) = z_{11}a_{11} + z_{12}a_{12} + z_{22}a_{22}$$

tengliklar o‘rinli. Integrallash sohasi sifatida

$$S_2^+ = \{A \in S_2: a_{11} > 0,\ a_{12} > 0,\ a_{22} > 0\}$$

olinadi. Bu yerda $A > 0$ qisqa yozuvi uchta mustaqil elementning musbatligini anglatadi. Boshqacha aytganda, bu yerda A ning musbat aniqlanganligi nazarda tutilmayapti. S_2^+ da $dA = da_{11}\, da_{12}\, da_{22}$ o‘lchovdan foydalanamiz.

1-ta’rif. φ: $\mathrm{S}_2 \to S_2^+$ lokal integrallanuvchi matritsa-funksiya bo‘lsin. Uning simmetrik matritsaviy Laplas almashtirishi quyidagi

$$\mathcal{L}_Z\{\varphi(A)\} = \hat{\varphi}(Z) := \int_{S_2^+} e^{-\mathrm{Sp}(ZA)}\, \varphi(A)\, dA \tag{4}$$

formula bilan aniqlanadi. Yadro skalyar bo‘lganligi sababli, uning $\varphi(A)$ga nisbatan yozilish tartibi natijani o‘zgartirmaydi.

1-lemma. Biror $X_0 = \big(\eta_{ij} x_{ij}^{(0)}\big) \in S_2$ va $M = M^T > 0$ uchun $|\varphi(A)| \le M e^{\mathrm{Sp}(X_0 A)}$, $A \in S_2^+$ baho bajarilsin. U holda $\hat{\varphi}$ funksiya

$$\mathcal{D} = \left\{Z = \left(\eta_{ij} z_{ij}\right) \in S_2^* : \mathrm{Re} z_{11} > x_{11}^{(0)},\ \mathrm{Re} z_{12} > x_{12}^{(0)},\ \mathrm{Re} z_{22} > x_{22}^{(0)}\right\}$$

sohada mavjud va golomorfdir. Bu yerda $|\cdot|$ modul matritsa elementlarining modulidan tuzilgan matritsani bildiradi va taqqoslash munosabati mos matritsa elementalar bo'yicha qaraladi.

Isbot. $X = \left(\eta_{ij} \mathrm{Re} z_{ij}\right)$ deb olsak, u holda $\left|e^{-\mathrm{Sp}(ZA)} \varphi(A)\right| \le M e^{-\mathrm{Sp}\left((X - X_0)A\right)}$.

Shu sabab

$$|\hat{\varphi}(Z)| \le \frac{M}{\left(\mathrm{Re} z_{11} - x_{11}^{(0)}\right)\left(\mathrm{Re} z_{12} - x_{12}^{(0)}\right)\left(\mathrm{Re} z_{22} - x_{22}^{(0)}\right)}.$$

Har bir $K \Subset \mathcal{D}$ kompakt uchun o'ng tomonni hosil qilgan majoranta Z ga bog'liq bo'lmagan integrallanuvchi funksiya bilan chegaralanadi. Integral ostida z_{11}, z_{12} va z_{22} bo'yicha differensiallashda faqat, mos ravishda, $-a_{11}$, $-a_{12}$ va $-a_{22}$ ko'paytuvchilar paydo bo'ladi. Ular ham biroz kichikroq yarim tekislikda shu usul bilan majorantlanadi. Parametrga bog'liq integrallar uchun golomorflik teoremasidan [16], $\hat{\varphi} \in \mathcal{O}(\mathcal{D})$ ekanligi kelib chiqadi.

1-izoh. $\mathcal{D}$ sohadagi shartlar koordinatalar bo'yicha qo'yilgan. Ularni keyin ishlatiladigan $\mathrm{Re} W > RI^{(2)}$ operator tengsizligi bilan almashtirmasligimiz kerak. Chunki musbat aniqlangan $\mathrm{Re} W$ matritsaning diagonaldan tashqari elementi manfiy bo'lishi mumkin. Shuning uchun Laplas almashtirish integralining aniqlanish sohasi va dinamik yarim tekislikning kesishmasi alohida ko'rsatiladi.

Kvadratik matritsaviy dinamika. S_2^* fazoda $F(Z) = Z - Z^2$ akslantirishni qaraymiz. Z yuqoridagi normallashgan koordinatalarda berilsa,

$$F(Z) = \begin{pmatrix} z_{11} - z_{11}^2 - \frac{1}{4} z_{12}^2 & \frac{1}{2}[z_{12} - z_{12}(z_{11} + z_{22})] \\ \frac{1}{2}[z_{12} - z_{12}(z_{11} + z_{22})] & z_{22} - z_{22}^2 - \frac{1}{4} z_{12}^2 \end{pmatrix}.$$

Demak, $F(S_2^*) \subset S_2^*$ va $F(O) = O$, bunda O nol matritsadir.

2-lemma. F akslantirishning $Z \in S_2^*$ nuqtadagi Frechet differensiali

$$DF(Z)[H] = H - ZH - HZ, \qquad H \in S_2^*$$

formula bilan beriladi. Xususan, $DF(O) = \mathrm{Id}_{S_2^*}$.

Isbot. $H \in S_2^*$ uchun

$$F(Z + H) - F(Z) = H - ZH - HZ - H^2.$$

Bunda $H \mapsto H - ZH - HZ$ uzluksiz kompleks-chiziqli operator, qoldiq esa $R_Z(H) = -H^2$. Operator normasining ko'paytmaga nisbatan baholash xossasidan

$$\frac{\| R_Z(H) \|_2}{\| H \|_2} \le \| H \|_2 \to 0, \qquad H \to O,$$

kelib chiqadi. Bu aynan Frechet differensial ta'rifidagi qoldiq shartidir. $Z = O$ qo'yilsa, $DF(O)[H] = H$ bo'ladi.

2-izoh. $DF(O) = \mathrm{Id}_{S_2^*}$ uch o'lchovli kompleks fazodagi birlik operatordir. Θ bazisida uning matritsasi $I^{(3)}$ ga teng. Abel tenglamasida qatnashuvchi

$$I^{(2)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in S_2^*$$

esa siljishni ifodalovchi fazo elementi bo'lib, $\Theta^{-1}\left(I^{(2)}\right) = (1,0,1)$. Shu sababli, $I^{(3)}$ operator matritsasi bilan $I^{(2)}$ siljish matritsasi bir xil obyekt emas.

Shuningdek, $F(O) = O$ bo'lgani uchun O matritsa $F(Z) = Z - Z^2$ akslantirishning qo'zg'almas nuqtasidir. Biroq S_2^* kompleks simmetrik matritsalar fazosida $F(Z) = Z$ tenglama $Z^2 = O$ shartga teng kuchli bo'lib, bu shartni qanoatlantiruvchi noldan farqli kompleks simmetrik matritsalar mavjud. Shu bois O ushbu fazoda yakkalangan qo'zg'almas nuqta emas. Bu esa bir o'zgaruvchili holdagi $f(z) = z - z^2$ akslantirish uchun 0 nuqta yakkalangan qo'zg'almas nuqta bo'lishi, matritsaviy analog olganda, farqli bo'lishini bildiradi.

Masalan,

$$\mho = \begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix} \ne O, \qquad \mho^2 = O,$$

shuning uchun har bir $t \in \mathbb{C}$ uchun $F(t\mho) = t\mho$. Demak, klassik Leau-Fatu teoremasini O ga yakkalangan parabolik nuqta haqidagi natija sifatida bevosita tatbiq qilib bo'lmaydi. Quyidagi natijalar singular matritsalarni chiqarib tashlaydigan maxsus invariant tortuvchi sohada bajariladi.

Teskarilanuvchi Z matritsalar uchun $W = Z^{-1}$ almashtirish kiritamiz. $F(Z) = Z(I_2 - Z)$ bo‘lganligi va bu ikki ko‘paytuvchi o‘zaro kommutativligi sabab

$$F(Z)^{-1} = \left(I^{(2)} - Z\right)^{-1} Z^{-1} = W^2\left(W - I^{(2)}\right)^{-1}.$$

Shu tariqa W koordinatadagi akslantirish

$$G(W) := W^2\left(W - I^{(2)}\right)^{-1} = W + I^{(2)} + \left(W - I^{(2)}\right)^{-1}$$

ko‘rinishga ega. Agar

$$W = \begin{pmatrix} w_{11} & w_{12}/2 \\ w_{12}/2 & w_{22} \end{pmatrix}, \qquad D_W = (w_{11} - 1)(w_{22} - 1) - \frac{1}{4}w_{12}^2,$$

bo‘lsa, u holda

$$G(W) = \begin{pmatrix} w_{11} + 1 + \dfrac{w_{22} - 1}{D_W} & \dfrac{1}{2}\left(w_{12} - \dfrac{w_{12}}{D_W}\right) \\ \dfrac{1}{2}\left(w_{12} - \dfrac{w_{12}}{D_W}\right) & w_{22} + 1 + \dfrac{w_{11} - 1}{D_W} \end{pmatrix}.$$

$W^* = \overline{W}^{\mathrm{T}}$ deb belgilab,

$$\mathrm{Re}W = \frac{W + W^*}{2}, \qquad \mathcal{H}_R^* = \{W \in S_2^* : \mathrm{Re}W > RI^{(2)}\}, \qquad R > 1,$$

matritsaviy o‘ng yarim tekislikni aniqlaymiz. Bu yerdagi tengsizlik Ermit matritsalarning musbat aniqlanganlik tartibida tushuniladi.

3-lemma. $\mathcal{H}_R^*$ soha G ga nisbatan oldinga invariant. $W_0 = W \in \mathcal{H}_R^*$ va $W_n = G^n(W)$ bo‘lsa,

$$\mathrm{Re}W_n > \mathrm{Re}W + nI^{(2)} > (R + n)I^{(2)}, \qquad \frac{W_n}{n} \to I_2. \tag{5}$$

Mos ravishda $Z = W^{-1}$ va $Z_n = F^n(Z) = W_n^{-1}$ uchun

$$Z_n \to O, \qquad nZ_n \to I^{(2)}. \tag{6}$$

Isbot. $W = A$ uchun $\mathrm{Re}A > (R - 1)I^{(2)} > O$. Shuning uchun A teskarilanuvchi va

$$\mathrm{Re}(A^{-1}) = (A^*)^{-1}(\mathrm{Re}A)A^{-1} > O.$$

Natijada

$$\mathrm{Re}G(W) = \mathrm{Re}W + I^{(2)} + \mathrm{Re}\left(W - I^{(2)}\right)^{-1} > \mathrm{Re}W + I^{(2)}.$$

Induksiya birinchi tengsizlikni beradi. Bundan tashqari,

$$\| \left(W_n - I^{(2)}\right)^{-1} \|_2 \leq \frac{1}{R + n - 1}.$$

$W_{n+1} = W_n + I^{(2)} + \left(W_n - I^{(2)}\right)^{-1}$ tenglikni yig‘ib, ushbu $W_n = W + nI^{(2)} + \sum_{j=0}^{n-1}\left(W_j - I^{(2)}\right)^{-1}$ munosabatni olamiz. Oxirgi yig‘indining normasi $O(\log n)$ dan oshmaydi, shu sabab $\frac{W_n}{n} \to I^{(2)}$.

Teskari matritsa amalining uzluksizligidan $nW_n^{-1} \to I^{(2)}$ va $W_n^{-1} \to O$ kelib chiqadi. ■

Abel tenglamasidagi qoldiq funksiya. $W \in \mathcal{H}_R^*$ bo‘lsa, W ning spektri ochiq o‘ng yarim tekislikda yotadi. Shu sabab manfiy haqiqiy yarim o‘q bo‘yicha kesilgan tekislikda aniqlangan bosh tarmoq orqali $\mathrm{Log}W$ bosh matritsaviy logarifmi mavjud va W ga golomorf bog‘liq. Taqribiy koordinatani

$$\Phi_0(W) = W - \mathrm{Log}W \tag{7}$$

deb olamiz.

4-lemma. Φ_0 ning matritsaviy Abel tenglamasidagi qoldiq funksiyasi

$$\Delta(W) := \Phi_0\left(G(W)\right) - \Phi_0(W) - I^{(2)} \tag{8}$$

quyidagi ko‘rinishga ega:

$$\Delta(W) = \left(W - I^{(2)}\right)^{-1} - \mathrm{Log}\left(W\left(W - I^{(2)}\right)^{-1}\right). \tag{9}$$

Agar $\alpha(W) = \lambda_{\min}(\mathrm{Re}W) > 1$ bo‘lsa, W ga bog‘liq bo‘lmagan $C > 0$ son uchun

$$\| \Delta(W) \|_2 \leq \frac{C}{(\alpha(W) - 1)^2} \tag{10}$$

baholash o‘rinli bo‘ladi.

Isbot. $G(W) = W^2\left(W - I^{(2)}\right)^{-1}$, W va $W\left(W - I^{(2)}\right)^{-1}$ bitta W matritsaning golomorf funksiyalari bo‘lgani uchun o‘zaro kommutativ. Bundan tashqari, quyidagi munosabat o‘rinli: $W\left(W - I^{(2)}\right)^{-1} = I^{(2)} + \left(W - I^{(2)}\right)^{-1}$.

Frullani formulasining matritsaviy funksional hisobdagi ko‘rinishidan [17], Δ funksiya uchun ushbu

$$\Delta(W)=\int_0^{\infty} e^{-tW}\left(e^t-\frac{e^t-1}{t}\right)dt$$

tenglik o‘rinli bo‘ladi. Qavs ichidagi funksiya $t\to 0$ da $t/2+O(t^2)$ ga teng va barcha $t>0$ lar uchun uning moduli Cte^t bilan chegaralanadi. $\| e^{-tW} \|_2 \le e^{-\alpha(W)t}$ tengsizlikdan

$$\| \Delta(W) \|_2 \le C\int_0^{\infty} t\, e^{-(\alpha(W)-1)t}\, dt=\frac{C}{(\alpha(W)-1)^2}$$

kelib chiqadi. ■

Matritsaviy Fatu koordinatasining qurilishi. Qoldiqni bartaraf etuvchi funksiyani

$$H(W):=\sum_{n=0}^{\infty}\Delta\left(G^n(W)\right),\qquad W\in\mathcal{H}_R^*, \tag{11}$$

qator orqali aniqlaymiz.

5-lemma. Yuqoridagi qator $\mathcal{H}_R^*$ ning har bir kompakt qismida tekis yaqinlashadi. H funksiya $\mathcal{H}_R^*$ da golomorf va

$$H\big(G(W)\big)=H(W)-\Delta(W)$$

tenglikni qanoatlantiradi.

Isbot. $W_n=G^n(W)$ deb olamiz. 3-lemmadan $\alpha(W_n)>\alpha(W)+n$ munosabat o‘rinli. 4-lemmadan esa $\| \Delta(W_n) \|_2 \le \frac{C}{(\alpha(W)+n-1)^2}$ tengsizlik kelib chiqadi.

Har bir $K \Subset \mathcal{H}_R^*$ uchun $\inf_{W\in K}\alpha(W)>R$ bo‘lganligi sabab, o‘ng tomondagi yaqinlashuvchi sonli qator K da yagona majoranta vazifasini bajaradi. Veyershtrass alomati qatorning normal yaqinlashishini, golomorf funksiyalar qatorlari haqidagi teorema esa H ning golomorfligini beradi [16]. Qator umumiy hadini bittaga siljitish orqali $H\big(G(W)\big)=\sum_{n=0}^{\infty}\Delta\left(G^{n+1}(W)\right)=H(W)-\Delta(W)$ tenglik olinadi. ■

1-teorema. $R>1$ bo‘lsin. U holda

$$\Phi(W)=W-\mathrm{Log}W+\sum_{n=0}^{\infty}\Delta\left(G^n(W)\right),\qquad W\in\mathcal{H}_R^*, \tag{12}$$

formula bilan aniqlangan $\Phi:\mathcal{H}_R^*\to S_2^*$ funksiya golomorf bo‘lib,

$$\Phi\big(G(W)\big)=\Phi(W)+I^{(2)} \tag{13}$$

matritsaviy Abel tenglamasini qanoatlantiradi.

Isbot. Abel tenglamasining yechimini $\Phi=\Phi_0+H$ ko‘rinishda izlaymiz. Bu yerda Φ_0 Abel funksional tenglamasining qoldiq funksiyasi bo‘lib, quyidagicha aniqlanadi:

$$\Phi_0\big(G(W)\big)=\Phi_0(W)+I^{(2)}+\Delta(W) \tag{14}$$

5-lemmaga ko‘ra,

$$H\big(G(W)\big)=H(W)-\Delta(W). \tag{15}$$

(14) va (15) tengliklar qo‘shilsa,

$$\Phi_0\big(G(W)\big)+H\big(G(W)\big)=\Phi_0(W)+I^{(2)}+\Delta(W)+H(W)-\Delta(W)$$
$$\Phi\big(G(W)\big)=\Phi_0(W)+I^{(2)}+H(W)$$
$$\Phi\big(G(W)\big)=\Phi(W)+I^{(2)}$$

hosil bo‘ladi. ■

Endi ushbu $\mathcal{P}_R=\{Z\in S_2^*\cap GL_2(\mathbb{C}):Z^{-1}\in\mathcal{H}_R^*\}$ to‘plamni qaraymiz. Bu yerda $GL_2(\mathbb{C})=\{Z\in\mathbb{C}^{2\times 2}:\det Z\neq 0\}$ to‘plam 2×2 teskarilanuvchi kompleks matritsalar to‘plami. $\mathcal{H}_R^*$ qavariq soha, $Z\mapsto Z^{-1}$ teskari matritsaga o‘tish akslantirishi esa $GL_2(\mathbb{C})$ da bigolomorf bo‘lganligi sababli, $\mathcal{P}_R$ ochiq va bog‘lamli to‘plam bo‘lib, soha tashkil qiladi. $tI^{(2)}\in\mathcal{P}_R$ tenglik $0<t<R^{-1}$ uchun bajariladi. Demak, $\mathcal{P}_R$ bo‘sh emas va O uning chegarasiga tegishli.

6-lemma. $\Psi(Z)=\Phi(Z^{-1})$ funksiya $\mathcal{P}_R$ da golomorf bo‘lib,

$$\Psi\big(F(Z)\big)=\Psi(Z)+I^{(2)}$$

tenglikni qanoatlantiradi. Bundan tashqari, $F(\mathcal{P}_R)\subset\mathcal{P}_R$ va har bir $Z\in\mathcal{P}_R$ uchun $F^n(Z)\to O$.

Isbot. $F(Z)^{-1}=G(Z^{-1})$ va $G(\mathcal{H}_R^*)\subset\mathcal{H}_R^*$ bo‘lganligi sabab $F(Z)\in\mathcal{P}_R$.

1-teoremaga ko‘ra,

$$\Psi\big(F(Z)\big)=\Phi(F(Z)^{-1})=\Phi\big(G(Z^{-1})\big)=\Phi(Z^{-1})+I^{(2)}=\Psi(Z)+I^{(2)}.$$

Yaqinlashishi $F^n(Z)=G^n(Z^{-1})^{-1}\to O$ munosabatdan kelib chiqadi. ■

Shu ma’noda $\mathcal{P}_R$ to‘la rangli invariant tortuvchi soha, Ψ esa matritsaviy Abel tenglamasining golomorf yechimi sifatidagi Fatu koordinatasidir.

2-teorema. $\mathrm{Re}W\succ I^{(2)}$ bo‘lsin. U holda

$$\Delta(W) = \left(W - I^{(2)}\right)^{-1} - \mathrm{Log}\left(W\left(W - I^{(2)}\right)^{-1}\right)$$

qoldiq funksiya quyidagi Laplas tipidagi integral tasvirga ega:

$$\Delta(W) = \int_0^{\infty} e^{-t(W-I^{(2)})}\left(1 - \frac{1-e^{-t}}{t}\right) dt.$$

Mazkur integral absolut yaqinlashadi va $\Delta(W)$ funksiya $\mathrm{Re}W \succ I^{(2)}$ sohada golomorfdir.

Isbot. $X = W - I^{(2)}$ deb olamiz. $\mathrm{Re}W \succ I^{(2)}$ bo'lganligi uchun $\mathrm{Re}X \succ 0$. Shuning uchun

$$X^{-1} = \int_0^{\infty} e^{-tX}\, dt.$$

Bundan tashqari, $s \in [0,1]$ uchun $X + sI^{(2)}$ teskari matritsaga ega va matritsaviy logarifmning integral formulasiga ko'ra,

$$\mathrm{Log}\left(I^{(2)} + X^{-1}\right) = \int_0^1 \left(X + sI^{(2)}\right)^{-1} ds.$$

$\mathrm{Re}\left(X + sI^{(2)}\right) \succ 0$ bo'lganligi sababli, $\left(X + sI^{(2)}\right)^{-1} = \int_0^{\infty} e^{-t(X+sI^{(2)})}\, dt = \int_0^{\infty} e^{-tX} e^{-st}\, dt$. Integrallash tartibini almashtirib,

$$\mathrm{Log}\left(I^{(2)} + X^{-1}\right) = \int_0^{\infty} e^{-tX}\frac{1-e^{-t}}{t}\, dt.$$

Endi $I^{(2)} + X^{-1} = W(W-I)^{-1}$ va $X = W - I^{(2)}$ ekanini hisobga olsak,

$$\Delta(W) = \int_0^{\infty} e^{-t(W-I^{(2)})}\left(1 - \frac{1-e^{-t}}{t}\right) dt.$$

$\mathrm{Re}\left(W - I^{(2)}\right) \succ 0$ bo'lgani uchun matritsaviy eksponenta eksponensial ravishda kamayadi. Shu sababli integral absolut yaqinlashadi. Integral ostidagi funksiya W bo'yicha golomorf va integral kompakt qism to'plamlarda tekis yaqinlashganligi sababli $\Delta(W)$ ham $\mathrm{Re}W \succ I^{(2)}$ sohada golomorfdir. ■

3-teorema. $R > 1$ bo'lsin va $\mathcal{H}_R^{\star} = \{W \in S_2^{\star} : \mathrm{Re}W \succ RI^{(2)}\}$ soha G akslantirishga nisbatan oldinga invariant bo'lsin, ya'ni $G(\mathcal{H}_R^{\star}) \subset \mathcal{H}_R^{\star}$. U holda

$$\Phi(W) = W - \mathrm{Log}W + \sum_{n=0}^{\infty}\int_0^{\infty} e^{-t(G^n(W)-I^{(2)})}\left(1 - \frac{1-e^{-t}}{t}\right) dt$$

qator $\mathcal{H}_R^{\star}$ sohaning har bir kompakt qism to'plamida tekis yaqinlashadi va Φ golomorf funksiya bo'ladi. Bundan tashqari,

$$\Phi\left(G(W)\right) = \Phi(W) + I^{(2)}$$

Abel funksional tenglamasi bajariladi.

Isbot. $\mathcal{H}_R^{\star}$ sohaning G akslantirishga nisbatan oldinga invariantligi va $\mathrm{Re}G(W) \succ \mathrm{Re}W + I^{(2)}$ munosabatdan induksiya bo'yicha

$$\mathrm{Re}G^n(W) \succ \mathrm{Re}W + nI^{(2)}$$

kelib chiqadi. $\alpha(W) = \lambda_{\min}(\mathrm{Re}W)$ deb olsak, 3-teoremadagi integral tasvirdan va (10) baholash munosabatidan

$$\| \Delta\left(G^n(W)\right) \| \leq \frac{1}{2(\alpha(W) + n - 1)^2}$$

tengsizlik hosil bo'ladi. Shu sababli $\sum_{n=0}^{\infty}\Delta\left(G^n(W)\right)$ qator $\mathcal{H}_R^{\star}$ ning har bir kompakt qismida tekis yaqinlashadi. Har bir $\Delta\left(G^n(W)\right)$ golomorf bo'lgani uchun Veyershtrass teoremasiga ko'ra qator yig'indisi ham golomorfdir [6]. $W - \mathrm{Log}W$ $\mathcal{H}_R^{\star}$ da golomorf bo'lganidan $\Phi(W)$ ham golomorf bo'ladi.

Endi $\Delta(W)$ ning ta'rifi va $G(W) = W + I^{(2)} + \left(W - I^{(2)}\right)^{-1}$ tenglikdan

$$G(W) - \mathrm{Log}G(W) = W - \mathrm{Log}W + I^{(2)} + \Delta(W)$$

kelib chiqadi. Bundan tashqari,

$$\sum_{n=0}^{\infty}\Delta\left(G^{n+1}(W)\right) = \sum_{n=0}^{\infty}\Delta\left(G^n(W)\right) - \Delta(W).$$

Ushbu tengliklarni $\Phi\left(G(W)\right)$ ifodasiga qo'ysak,

$$\Phi\left(G(W)\right) = \Phi(W) + I^{(2)}$$

hosil bo'ladi. Demak, Φ Abel funksional tenglamasini qanoatlantiradi. ■

Xulosa. $S_2^* \cong \mathbb{C}^3$ bo‘lib, simmetrik matritsa argumentli matritsa-funksiya bilan ishlash uch kompleks o‘zgaruvchili vektor-funksiyani o‘rganishga ekvivalentdir. Matritsalar bilan ishlash shunisi bilan qulayki, $F(Z) = Z - Z^2$, $W = Z^{-1}$ va $G(W) = W + I^{(2)} + \left(W - I^{(2)}\right)^{-1}$ kabi formulalar matritsaviy yozuvda algebraik jihatdan ixcham saqlanadi. Agar koordinatalarga bevosita o‘tsak, u holda o‘zaro bog‘langan uchta nochiziqli komponent hosil bo‘ladi. Shuning uchun S_2^* ishlash qulayroq $\mathbb{C}^3$ da ishlaganga qaraganda. Keyingi ilmiy izlanishlarimizda $Z = O$ nuqta atrofida golomorf bo‘lgan $F(Z) = Z + Z^2 + Z^3 + \cdots = Z\left(I^{(2)} - Z\right)^{-1}$ akslantirish hamda umumiyroq $F(Z) = Z + P_2(Z) + P_3(Z) + \cdots$ ko‘rinishidagi matritsaviy akslantirishlar uchun Fatu koordinatalarini qurish hamda ularning analitik va asimptotik xossalarini o‘rganishni o‘zimizga maqsad qilib qo‘yganmiz.

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KLASSIK VA ZAMONAVIY FUNKSIONAL TENGLAMALAR HAMDA ULARNING SODDA YECHISH USULLARI

Annotatsiya. *Ushbu maqolada klassik va zamonaviy funksional tenglamalarni yechishning samarali hamda sodda usullari yoritilgan. Tadqiqotda Koshi funksional tenglamasi, matematik induksiya usuli, funksiyaning uzluksizligi va zichlik xossasi, shuningdek, involutsiya tushunchalariga asoslangan teoremalar hamda lemmalar keltirib o‘tilgan va ularning isbotlari berilgan. Nazariy ma’lumotlarni mustahkamlash*