$$\beta_\eta(\alpha) = \left\| S_h l_{i,j}^{*\eta,\alpha} - m_{y,i,j}^{\delta} \right\|_{U^2},$$

$$\rho_\eta(\alpha) = \beta_\eta(\alpha) - \left(\delta + h\sqrt{\gamma_\eta(\alpha)}\right)^2 - \mu_\eta^2.$$

Bu yerda $\alpha > 0$ da qayd qilingan

$$\mu_\eta = \inf_{l_{i,j}^* \in D} \left\| S_h l_{i,j}^* - m_{y,i,j}^{\delta} \right\|_U, G^\alpha \left[l_{i,j}^* \right]$$

A.N.Tixonov funksiyasining ekstremali – D ko‘paytmasidagi taxminiy ma’lumotlar bilan (16) tenglamaning mos kelmaslik o‘lchamidir.

Xulosa. Muntazamlashtirish parametrini aniqlash uchun, ko‘pincha, xatolik uslubi yoki xatolikning umumlashgan prinsipidan foydalaniladi [6, 7, 9]. Xatolik uslubidan foydalanishda $\alpha(\delta)$ muntazamlashtirish parametri

$$\beta_\delta(\alpha) = \left\| S l_{i,j}^{*\delta} - m_{y,i,j}^{\delta} \right\|_U = \delta$$

tenglamaning ildizi kabi tanlanadi [7].

Muntazamlashtirish parametri α ni aniqlashda, ko‘pincha, quyidagi

$$\beta_\delta(\alpha) - \left(\delta + h\sqrt{\gamma_\eta(\alpha)}\right)^2 = 0$$

ko‘rinishdagi xatolikning umumlashgan algoritmidan foydalaniladi.

Foydalanilgan adabiyotlar ro‘yxati:

[1]. Бакушинский А.Б., Гончарский А.В. Интерактивные методы решения некорректных задач. М., “Наука”, 1989, 128 с.

[2]. Егупов Н.Д., Пупков К.А. Методы классической и современной теории автоматического управления. Учебник в 5 томах, М., издателство МГТУ им. Н.Э.Баумана, 2004.

[3]. Kodirova F., Kholkhodjaev B.A., Kuralov B.A., Xudoyberdiev A. “Advanced control systems for dynamic engineering applications”. AIP Conf. Proc. 3304, 030057 (2025). (Scopus) https://doi.org/10.1063/5.0270490

[4]. Холходжаев Б.А. Алгоритмы восстановления входных воздействий динамических систем в условиях неопределенности. «Technical science and innovation», № 4/2020. Ташкент, 2020, с. 150–154.

[5]. Мирошник И.В., Никифоров В.О., Фрадков А.Л. Нелинейное и адаптивное управление сложными динамическими системами. СПб., “Наука”, 2000.

[6]. Zayniddinov B., Kholkhodjayev.B.A., Abdishukurov Sh., Zayniddinova Z. AIP Conference Proceedings, 2023, 2612, 020004. (2023), p. 1–7. (Scopus) https://doi.org/10.1063/5.0116887

[7]. Короткий А.И. Восстановление управлений и параметров динамических систем при неполной информаци. Изв. высш. учеб. заведений, 1998, № 11 (438), с. 109–120.

[8]. Афанасев В.Н. Динамические системы управления с неполной информацей. Алгоритмическое конструирование. Изд-во “КомКнига”, 2007.

[9]. B.Kholhodjaev, B.A.Kuralov, K.Daminov. (2023) Block diagram and Mathematical model of an invariant system/ «Technical science and innovation» ТГТУ, № 2 (16), 2023, с. 186–194.

[10]. Еремин Е.Л., Теличенко Д.А., Семичевская Н.П., Чепак Л.В, Шеленок Е.А. Управление техническими системами в условиях неопределенности. Благовещенск, Амурский гос. ун-т, 2014, 211 с.

Husenov Behzod Erkin o‘g‘li (Buxoro davlat universiteti; b.e.husenov@buxdu.uz),
Muhammadova Mastura Farhod qizi (Buxoro davlat universiteti; masturamuhammadova@mail.ru)

$A(z)$-ANALITIK FUNKSIYALARNING LOKAL XOSSALARI

Annotatsiya. *Ushbu maqolada $A(z)$ funksiyani antianalitik bo‘lgan holda $A(z)$ – analitik funksiyalarni qaraymiz. Asosiy natijalar sifatida lokal $A(z)$ – analitik funksiya tushunchasi kiritildi, shuningdek $A(z)$ – analitik funksiyalarni ko‘phadlarning yig‘indisidan tuzilgan darajali qator orqali aproksimatsiyalash (yaqinlashtirish) masalasining yechimi $A(z)$ – analitik funksiyalar uchun Runge teoremasi or-qali berildi.*

Kalit so‘zlar: *$A(z)$ – analitik funksiya, $A(z)$ – lemniskata, $A(z)$ – analitik funksiyalar uchun Teylor va Loran qatorlari, $A(z)$ – analitik funksiyalarni qatorlar orqali aproksimatsiyalash.*

ЛОКАЛЬНЫЕ СВОЙСТВА *A*(*z*)-АНАЛИТИЧЕСКИХ ФУНКЦИЙ

Аннотация. *Мы рассматриваем* $A(z)$ *– аналитические функции в случае, когда* $A(z)$ *антианалитическая функция. В качестве основных результатов введено понятие локальной* $A(z)$ *– аналитической функции, а также решение задачи аппроксимации* $A(z)$ *– аналитических функций степенными рядами, составленными из сумм многочленов, получено с использованием теоремы Рунге для* $A(z)$ *– аналитических функций.*

Ключевые слова: $A(z)$ *– аналитическая функция;* $A(z)$ *– лемниската; ряд Тейлора и Лорана для* $A(z)$ *– аналитических функций, аппроксимация* $A(z)$ *– аналитических функций с помощью рядов.*

LOCAL PROPERTIES OF *A*(*z*)-ANALYTIC FUNCTIONS

Annotation. *We consider* $A(z)$ *– analytic functions in case when* $A(z)$ *is antianalytic function. As the main results, the concept of a local* $A(z)$ *– analytic function is introduced, and the solution to the problem of approximating* $A(z)$ *– analytic functions by power series composed of sums of polynomials is obtained using Runge's theorem for* $A(z)$ *– analytic functions.*

Keywords: $A(z)$ *– analytic function;* $A(z)$ *– lemniscate; Taylor and Laurent series for* $A(z)$ *– analytic functions; approximation of* $A(z)$ *– analytic functions by means of series.*

1. Kirish. 1.1. *A*(*z*)-analitik funksiyalar to‘g‘risida dastlabki tushunchalar. Kvazikonform akslantirishlar nazariyasi bilan bog‘liq bo‘lgan ushbu

$$\frac{\partial f}{\partial \overline{z}} = A(z)\frac{\partial f}{\partial z} \tag{1}$$

Beltrami tenglamasining yechimini qaraylik. Bu yerda $A(z)$ funksiya o‘lchovli va qaralayotgan $D \subset \mathbf{J}$ sohaning deyarli barcha nuqtalarida $|A(z)| \le c$ shartni qanoatlantiradi, bu yerda $c < 1$. Adabiyotlarda (1) tenglamaning yechimi *A*(*z*)-*analitik funksiya* deb ataladi.

Aytaylik, *A*(*z*) antianalitik funksiya va $\forall z \in D$ uchun $|A(z)| \le c < 1$ o‘rinli bo‘lsin. Shunday qilib, antianalitik funksiyalar cheksiz silliq bo‘lganligidan $O_A(D) \subset C^{\infty}(D)$ ekanligi kelib chiqadi [2].

1-teorema (Koshi teoremasining analogi, [3]). *Faraz qilaylik,* $D \subset \mathbf{J}$ *soha bo‘lakli silliq chegaradan iborat bo‘lsin. Agar* $f \in O_A(D) \cap C(\overline{D})$ *bo‘lsa, u holda*

$$\int_{\partial D} f(z)\left(dz + A(z)d\overline{z}\right) = 0$$

bo‘ladi.

$A(z)$ – analitik funksiyalarni o‘rganishda, $A(z)$ – antianalitik funksiya bo‘lishida Koshi yadrosi katta ahamiyatga ega:

$$K(z,\zeta) = \frac{1}{2\pi i}\frac{1}{z-\zeta+\overline{\int\limits_{\gamma(z,\zeta)} \overline{A(\tau)}d\tau}}, \tag{2}$$

bu yerda $\gamma(z,\zeta) - \zeta$, $z \in D$ nuqtalarni tutashtiruvchi egri chiziq yoki ζ nuqtadan z nuqtaga boruvchi ixtiyoriy yo‘lni $\gamma(z,\zeta)$ bilan belgilaymiz. Shunday qilib, D soha bir bog‘lamli va $\overline{A}(z)$ – golomorf funksiya bo‘lsa, u holda $I(z) = \int\limits_{\gamma(\zeta,z)} \overline{A}(\tau)d\tau$ integrallash yo‘liga bog‘liq bo‘lmaydi, u dastlabki holat $I'(z) = \overline{A}(z)$ bilan to‘g‘ri keladi [5].

Agar $D \subset \mathbf{J}$ soha bo'lsa, u holda quyidagi teorema o'rinli:

2-teorema [5]. *$K(z,\zeta)$ yadro $z=\zeta$ dan tashqari boshqa nuqtalarda $A(z)$ – analitik funksiya bo'ladi, ya'ni $K \in O_A(D \setminus \zeta)$. Shuningdek, $K(z,\zeta)$ uchun $z=\zeta$ nuqta birinchi tartibli qutb nuqta bo'ladi.*

1-tasdiq [5]. *Agar D soha qavariq bo'lmasa, faqatgina bir bog'lamli bo'lsa, u holda $\psi(z,\zeta)=z-\zeta+\overline{\int_{\gamma(z,\zeta)} A(\tau)d\tau}$ funksiya ham D sohada bir qiymatli aniqlanadi, ammo u boshqa yakkalangan nollarga ζ: $\psi(z,\zeta)\neq 0$, $z \in P=\{\zeta,\zeta_1,\zeta_2,\zeta_3,...\}$ ega bo'lishi mumkin. Biroq $\psi \in O_A(D)$, $\psi(z,\zeta)\neq 0$, $z \notin P$ da va $K(z,\zeta)$ yadro $D \setminus P$ da oddiy qutb P to'plamning nuqtalarida $A(z)$ – analitik funksiya bo'ladi.*

Aytaylik, $D \subset \mathbf{J}$ qavariq soha bo'lsin. $\psi(z,\xi)=z-\xi+\overline{\int_{\gamma(\xi,z)} \overline{A}(\tau)d\tau} \in O_A(D)$ funksiya ichki akslantirishni amalga oshiradi. Xususan, $L(a,r)=\{z \in D : |\psi(a,z)|<r\}$ to'plam D da ochiq to'plamni ifodalaydi. Yetarlicha kichik $r>0$ uchun bu to'plam D da kompakt yotadi va nuqtani o'z ichi-ga oladi. Bu to'plamni markazi a bo'lgan *A(z)-lemniskata* deyiladi va $L(a,r)$ ko'rinishda belgilanadi [5].

3-teorema (Koshi formulasining analogi, [4]). *Faraz qilaylik, $D \subset \mathbf{J}$ – qavariq soha va $G \subset D - \partial G$ chegarasi bo'lakli silliq soha bo'lsin. U holda ixtiyoriy $f \in O_A(D) \cap C(\overline{D})$ funksiya uchun quyidagi formula o'rinli*

$$f(z)=\int_{\partial G} K(z,\xi)f(\xi)\left(d\xi+A(\xi)d\overline{\xi}\right),\quad z \in G. \tag{3}$$

1.2. *A(z)*-analitik funksiyalar uchun Teylor va Loran qatorlari. Aytaylik, $D \subset \mathbf{J}$ bo'lsin. Avvalo, $A(z)$ – *analitik funksiyalar uchun Teylor qatori* quyidagi ifodalanadi:

$$\sum_{j=0}^{\infty} c_j \psi^j(z,a), a \in D, \tag{4}$$

bu yerda $c_j = const$.

Qatorning yaqinlashish sohasi $L(a,r)=\{|\psi(z,a)|<r\}$ lemniskata bilan belgilasak, bu yerda yaqinlashish radiusi Koshi-Adamar formulasi yordamida aniqlanadi:

$$\frac{1}{r}=\overline{\lim_{j\to\infty}}\sqrt[j]{|c_j|}.$$

Ushbu qator uchun quyidagi teorema o'rinli:

4-teorema [5]. *Agar $f(z) \in O_A(L(a,r)) \cap C(\overline{L}(a,r))$ bo'lsa, unda $f(z)$ funksiyani $L(a,r)$ da Teylor qatori sifatida (4) ko'rinishida yoyish mumkin. Bu yerda koeffitsiyentlar quyidagicha topiladi:*

$$c_k=\frac{1}{2\pi i}\int_{\partial L(a,\rho)} \frac{f(\xi)}{\left[\psi(\xi,a)\right]^{k+1}}\left(d\xi+A(\xi)d\overline{\xi}\right), 0<\rho<r, k=0,1,2\ldots.$$

5-teorema (Loran teoremasining analogi, [5]). *$f(z)$ funksiya $\{L(a,R)\setminus L(a,r)\}$ halqada $A(z)$ – analitik bo'lsa, u holda ushbu funksiya ushbu halqada quyidagicha Loran qatoriga yoyiladi:*

$$f(z)=\sum_{k=-\infty}^{\infty}c_k\psi^k(z,a),$$

bu yerda c_k *– Teylor-Loran qatorining koeffitsiyentlari bo'lib, quyida formula orqali aniqlanadi:*

$$c_k=\frac{1}{2\pi i}\int\limits_{\partial L(a,\rho)}\frac{f(\xi)}{\left[\psi(\xi,a)\right]^{k+1}}\left(d\xi+A(\xi)d\overline{\xi}\right),r<\rho<R,k\in\text{ў}.$$

Shuningdek, (4) *qator halqaning ichida tekis yaqinlashadi.*

Koshi tengsizligi [5]. *Teylor-Loran qatori koeffitsiyentlari uchun quyidagi tengsizliklar bajariladi:*

$$\left|c_k\right|\le\frac{\max\limits_{z\in\partial L(a,\rho)}\left\{\left|f(z)\right|\right\}}{\rho^k},r<\rho<R,k\in\text{ў}$$

bu munosabat oddiy usulda

$$\int\limits_{\partial L(a,p)}\left|d\xi+A(\xi)d\overline{\xi}\right|=2\pi\rho$$

ifoda orqali kelib chiqadi.

2. Asosiy natijalar. 2.1. ***A*(*z*)-analitik funksiyalarning lokal xossalari.** Aytaylik, $D\subset\text{J}$ qavariq va $L(a,r)\subset\subset D$ kompakt to'plam berilgan bo'lsin.

1-ta'rif. *Agar* $f(z)\in O_A(D)$ *funksiya har bir* $a\in D$ *nuqta uchun*

$$\sum_{n=0}^{\infty}c_n\psi^n(z,a)$$

qator mavjud bo'lsa, u holda $f(z)$ *funksiya* D *sohada lokal* $A(z)-$*analitik funksiya deyiladi va bu funksiya qatorga yoyiladi:*

$$f(z)=\sum_{n=0}^{\infty}c_n\psi^n(z,a).$$

Quyidagi ichma-ich joylashgan kompakt to'plamlar oilasini qaraymiz:

$$\{G_n\}=\left\{L\left(a,\frac{nr}{n+1}\right)\right\}\subset\subset L(a,r),$$

bu yerda $n\in\text{Г}$. Shuningdek, chegaraviy to'plamni $\partial G_n=\Gamma_n$ kabi belgilaymiz. $f(z)\in O_A(L(a,r))\text{I}\ C(\overline{L}(a,r))$ funksiya uchun quyidagi Koshining integral formulasini qaraymiz:

$$f(z)=\int\limits_{\Gamma_{n+1}}f(\zeta)K(z,\zeta)\left(d\zeta+A(\zeta)d\overline{\zeta}\right),\qquad(5)$$

bu yerda $z\in\overline{G}_n$. Γ_{n+1} bo'yicha (5) integraldan kelib chiqadiki, bu integral yoyilgan alohida to'g'rilanuvchi yopiq Jordan egri chiziqlaridan olingan integrallarning yig'indisidan iborat. Agar $z\in\overline{G}_n$ bo'lsa, u holda bu nuqta G_{n+1} komponentali bitta to'plamning ichida joylashadi, uning chegarasini γ' bilan belgilaymiz. U holda bu egri chiziq bo'yicha olingan Koshi tipidagi integral (5) kabi ifodalanadi:

$$f(z)=\int\limits_{\gamma}f(\zeta)K(z,\zeta)\left(d\zeta+A(\zeta)d\overline{\zeta}\right),$$

qolgan G_{n+1} komponentali sohalarning chegarasi bo'yicha olingan integrallar Koshi teoremasidan (3-teorema) nolga teng bo'ladi.

Ixtiyoriy $\varepsilon > 0$ son olinganda (5) integral uchun shunday integrallar yig‘indisini topish mumkinligini ko‘rsatamizki,

$$S^{(n+1)}(z) = S^{(n+1)}(z,\varepsilon),$$

shart bajariladi, G_n to‘plamning z barcha nuqtalarida quyidagi tengsizlik bajariladi:

$$\left| f(z) - S^{(n+1)}(z,\varepsilon) \right| < \varepsilon. \tag{6}$$

Endi oxirgi tengsizlikni yanada o‘rinli bo‘lishi ko‘rsatish maqsadida $\forall \varepsilon' > 0$ son uchun $\exists \delta(\varepsilon') > 0$ tanlaymiz, $\forall \zeta', \zeta'' \in \Gamma_{n+1}$ uchun

$$\left| \zeta' - \zeta'' \right| < \delta(\varepsilon')$$

shartni qanoatlantiruvchi nuqtalar juftligida quyidagi munosabat o‘rinli bo‘ladi:

$$\left| f(\zeta') - f(\zeta'') \right| < \varepsilon'.$$

Yuqoridagilardan ko‘rinadiki, $f(z)$ funksiya Γ_{n+1}da uzluksiz. Keyin Γ_{n+1} egri chiziqni kichik σ_k yoylarga bo‘lamiz, bunda σ_k yoyning boshlang‘ich va tugash nuqtasi, mos ravishda, $\xi_k'^{(n+1)}$ va $\xi_k''^{(n+1)}$ iborat bo‘lib $\left(k = 1,2,\ldots,n_0(\varepsilon')\right)$, σ_k yoyning uzunligi bo‘lgan λ_k uchun quyidagi shart bajariladi:

$$\lambda_k < \min\left\{\delta(\varepsilon'), \varepsilon'\right\}.$$

σ_k yoyning chekli nuqtasi σ_{k+1} yoyning boshlang‘ich nuqtasi bo‘ladi; lekin k uchun bu shart buziladi, ya’ni unda σ_k va σ_{k+1} yoylar Γ_{n+1} ni tuzadigan turli chiziqlarda yotadi (umuman olganda, Γ_{n+1} ni bog‘liqsiz to‘plamligini unutmaylik). Nihoyat, $S^{(n+1)}(z)$ integral yig‘indini quyidagicha yoyning bo‘laklanishida tuzamiz:

$$S^{(n+1)}(z) = \frac{1}{2\pi i} \sum_{k=1}^{n_0(\varepsilon')} \frac{f\left(\zeta_k'^{(n+1)}\right)}{\psi\left(z, \zeta_k'^{(n+1)}\right)} \psi\left(\zeta_k''^{(n+1)}, \zeta_k'^{(n+1)}\right).$$

U vaqtda $\left| f(z) - S^{(n+1)}(z) \right|$ ayirma moduli uchun quyidagi ifodani olamiz:

$$\left| f(z) - S^{(n+1)}(z) \right| = \left| \int_{\Gamma_{n+1}} f(\zeta) K(z,\zeta)\left(d\zeta + A(\zeta) d\bar{\zeta}\right) - \frac{1}{2\pi i} \sum_{k=1}^{n_0(\varepsilon')} \frac{f\left(\zeta_k'^{(n+1)}\right)}{\psi\left(z, \zeta_k'^{(n+1)}\right)} \psi\left(\zeta_k''^{(n+1)}, \zeta_k'^{(n+1)}\right) \right| =$$

$$= \left| \sum_{k=1}^{n_0(\varepsilon')} \int_{\sigma_k} \left(f(\zeta) K(z,\zeta) - f\left(\zeta_k'^{(n+1)}\right) K\left(z, \zeta_k'^{(n+1)}\right) \right) \left(d\zeta + A(\zeta) d\bar{\zeta}\right) \right| =$$

$$= \left| \sum_{k=1}^{n_0(\varepsilon')} \int_{\sigma_k} K(z,\zeta) \left(f(\zeta) - f\left(\zeta_k'^{(n+1)}\right) + f\left(\zeta_k'^{(n+1)}\right) \frac{\psi\left(\zeta_k'^{(n+1)}, \zeta\right)}{\psi\left(\zeta_k'^{(n+1)}, z\right)} \right) \left(d\zeta + A(\zeta) d\bar{\zeta}\right) \right|.$$

Keyin faraz qilamiz, $\max\limits_{z \in \Gamma_k} \left| f(z) \right| = M_{n+1}$ bo‘lsin va har bir $\bar{G}_n$ va Γ_{n+1} orasidagi masofani $\rho_{n+1} > 0$ orqali belgilaymiz, barcha $z \in \bar{G}_n$ lar uchun quyidagi munosabat bajariladi:

$$\left| f(z) - S^{(n+1)}(z) \right| \le \sum_{n=1}^{n_0(\varepsilon')} \int_{\sigma_k} \left| \left(f(\zeta) - f\left(\zeta_k'^{(n+1)}\right) \right) K(z,\zeta) \right| \left| d\zeta + A(\zeta) d\bar{\zeta} \right| +$$

$$+\sum_{n=1}^{n_0(\varepsilon')}\int_{\sigma_k}\left|f\left(\zeta_k^{'(n+1)}\right)K(z,\zeta)\right|\left|\frac{\psi\left(\zeta_k^{'(n+1)},\zeta\right)}{\psi\left(\zeta_k^{'(n+1)},z\right)}\right|\left|d\zeta+A(\zeta)d\bar{\zeta}\right|<\frac{1}{2\pi}\left(\sum_{n=1}^{n_0(\varepsilon')}\varepsilon'\lambda_k\frac{\rho_{n+1}+M_{n+1}}{\rho_{n+1}^2}\right).$$

Nihoyat, $\sum_{n=1}^{n_0(\varepsilon')}\lambda_k=\Lambda_{n+1}$ belgilaymiz, bunda Λ_{n+1} barcha Γ_{n+1} egri chiziqlar uzunliklaridan iborat yig‘indi va bundan quyidagini olamiz:

$$\left|f(z)-S^{(n+1)}(z)\right|<\frac{\varepsilon'\Lambda_{n+1}}{2\pi\rho_{n+1}^2}\left(\rho_{n+1}+M_{n+1}\right).$$

Ma’lumki, ε' olinganda uning yetarlicha kichik qiymati berilgan ixtiyoriy $\varepsilon>0$ miqdordan ham kichik qilib tanlab olinadi.

Faraz qilaylik, $\varepsilon=\varepsilon_n$ bo‘lib, $n\to\infty$ da $\varepsilon_n\to 0$ bo‘lganda, ratsional funksiyalar $S^{(n+1)}(z)=S^{(n+1)}(z,\varepsilon_{n+1})$ ketma-ketligini olamiz, bunda bu ketma-ketlik har bir yopiq $\bar{G}_k$ to‘plamda $f(z)$ ga tekis yaqinlashadi va shuningdek, D sohaning ichida ham $f(z)$ ga tekis yaqinlashadi.

1-tasdiq. *Ixtiyoriy ochiq D to‘plam uchun va ixtiyoriy $f(z)\in O_A(D)\mid C(\bar{D})$ funksiya uchun shunday ratsional funksiyalar ketma-ketligi $R_n(z)=S^{(n+1)}(z)$ mavjud bo‘lib, D ning ichkarisida $f(z)$ ga tekis yaqinlashadi.*

1-lemma. *Aytaylik, E chegaralangan yopiq to‘plam va ζ nuqta E ning tashqarisida joylashsin. Shuningdek, $R(z)-$ratsional funksiya va ζ yagona qutb nuqtasi bo‘lsin: $R(z)=\frac{P(z)}{\psi^k(z,\zeta)}$ ($P(z)$ ko‘phadning darajasi k dan oshmasin). Agar ξ va ζ nuqtalar E sohaga qo‘shni bo‘lgan E_1 sohaga tegishli bo‘lsa, u holda ixtiyoriy $\varepsilon>0$ uchun $\tilde{R}(z)$ ratsional funksiya qurishimiz mumkin: $\tilde{R}(z)=\frac{\tilde{P}(z)}{\psi^k(z,\xi)}$ ($\tilde{P}(z)$ ning darajasi k dan oshmaydi) shundayki, $z\in E$ uchun $\left|\tilde{R}(z)-R(z)\right|<\varepsilon$ bajariladi.*

Isbot. Isbot uchun E_1 soha ichkarisida ζ va ξ nuqtalarni uzluksiz γ'' egri chiziq orqali tutashtiramiz va γ'' va E_1 soha orasidagi masofani $\rho>0$ orqali belgilaymiz. γ'' ni diametri $\frac{\rho}{2}$ dan kichik bo‘lgan $\sigma_1,\sigma_2,\ldots,\sigma_m$ yoylarga bo‘lamiz va nuqtalarning bo‘linishini $\zeta_0=\zeta,\zeta_1,\zeta_2,\ldots,\zeta_m=\xi$ kabi belgilaymiz va quyidagi ratsional funksiyani qaraymiz:

$$R_1(z)=\frac{P(z)}{\psi^k(z,\zeta)}\left(1-\left(\frac{\psi(\zeta,\zeta_1)}{\psi(z,\zeta_1)}\right)^{n_1}\right)^k=\frac{P_1(z)}{\psi^{n_1k}(z,\zeta_1)}$$

($P_1(z)$ ning darajasi $k+n_1k-k=n_1k$ dan oshmaydi). $z\in E$ uchun

$$\left|R_1(z)-R(z)\right|=\left|R(z)\right|\left|1-\left(\frac{\psi(\zeta,\zeta_1)}{\psi(z,\zeta_1)}\right)^{n_1}-1\right|\le M\left(\binom{k}{1}\left(\frac{\rho}{2}\right)^{n_1}+\ldots\right)<M\frac{2^k}{2^{n_1}}$$

munosabat o‘rinli bo‘ladi, bunda $M=\max_{z\in E}\left|R(z)\right|$.

Yetarlicha katta n_1 tanlab, $z\in E$ uchun quyidagi tengsizlik bajariladi:

$$\left|R_1(z)-R(z)\right|<\frac{\varepsilon}{m}.$$

Bu fikrni takrorlab, $m-$ qadamda kerakli $R_m(z)=R_m(z)$ funksiyani olamiz. ■

2.2. $A(z)$-analitik funksiyalar uchun Runge teoremasi. 6-teorema (Runge teoremasining analogi). *D chekli va qavariq soha, shuningdek, juft-jufti bilan umumiy nuqta bo'lmagan nuqtalardan tashkil topgan va ∞ nuqtani o'z ichiga olmaydigan to'plam bo'lsin. D sohada lokal $A(z)-$analitik bo'lgan har bir funksiya uchun $\{P_n(z)\}$ ko'phadlar ketma-ketligini qurishimiz mumkinki, bu ko'phad $f(z)$ funksiyaga tekis yaqinlashadi.*

Isbot. Isbot uchun har bir G_n komponentali to'plam olamiz, teoremadagi shartlar, ya'ni qavariq soha hosil qiladi, shunday komponentali to'plamning chegarasi yopiq Jordan yoylaridan iborat bo'lgan kamida bitta to'plam D sohaga tegishli bo'ladi. Bundan G_{n+1} ning chegarasi Γ_{n+1} ning har bir nuqtasi, xususan har bir $\zeta_k^{'(n+1)}$ nuqta Jordan yoylarini tutashtiradi, $\overline{G}_n$ bilan umumiy nuqtasi bo'lmaydi, aynan ξ nuqta bilan ustma-ust tushmaydi, shuningdek, bu nuqta $\overline{G}_n$ ni o'z ichiga oladigan $L(a,r_n)$ning tashqarisida joylashadi. Lemmadagi ratsional funksiyani

$$f\left(\zeta_k^{'(n+1)}\right)K\left(z,\zeta_k^{'(n+1)}\right)\psi\left(\zeta_k^{'(n+1)},\zeta_k^{''(n+1)}\right),\ k=1,2,\ldots,n_0^{(n+1)},$$

tatbiq etib, $\varepsilon=\dfrac{\varepsilon_n}{n_0^{(n+1)}}$ bo'lganda, $S_n(z)$ ratsional funksiyani quramiz, ya'ni yagona qutb ξ nuqtasi bo'lgan: $S_n(z)=\dfrac{P(z)}{\psi^{n}(z,\xi)}$ ($P(z)$ ning darajasi n dan oshmaydi) $\overline{G}_n$ to'plamning barcha nuqtalarida

$$\left|S_n(z)-S^{(n+1)}(z)\right|<\varepsilon_n$$

tengsizlikni qanoatlantiradi, shuningdek,

$$\left|f(z)-S_n(z)\right|<2\varepsilon_n$$

tengsizlikni ham qanoatlantiradi.

Ammo $S_n(z)$ funksiya $L(a,r_n)$ lemniskatada $A(z)-$analitik bo'ladi va shuningdek, $\overline{G}_n$ to'plamda $L(a,r_n)$ kompakt yotadi, uni darajali qatorga yoyish mumkin. Yetarlicha uzun kesma olsak, $P_n(z)$ darajali qator barcha $z\in\overline{G}_n$ larda quyidagi tengsizlik bajariladi:

$$\left|f(z)-P_n(z)\right|<3\varepsilon_n.$$

$\{P_n(z)\}$ ketma-ketlik teoremaning barcha shartlarini qanoatlantiradi. ■

1-natija. *Aytaylik, B chegaralangan mukammal to'plam, uning to'ldiruvchisi $\mathbf{J}\setminus B=E$ bo'lsin, E to'plam B nisbatan kompleks tekislikka kengaysin, bog'lamli, ya'ni soha hosil qilsin. U holda ixtiyoriy $f(z)$ funksiya B da lokal $A(z)-$analitik va ixtiyoriy $\varepsilon>0$ uchun shunday $Q(z)$ ko'phad mavjud bo'lib, $z\in B$ lar uchun quyidagi munosabat o'rinli bo'ladi:*

$$\left|f(z)-Q(z)\right|<\varepsilon.$$

Isbot. Har bir $a\in B$ nuqta uchun shunday $L(a,\rho)$ lemniskata mavjudki, $L(a,\rho)$ ning tashqarisida joylashgan B to'plamning barcha nuqtalarida $f(z)$ funksiya $\psi(z,a)$ ning darajali ko'rinishidagi qatorga yoyiladi. B to'plamning ochiq qoplamasidan $L(a,\rho)=\left\{\left|\psi(z,a)\right|<\dfrac{\rho}{2}\right\}$ chekli qoplamasini tanlaymiz: $L(a_1,\rho)$, $L(a_2,\rho)$,..., $L(a_p,\rho)$. Bu lemniskatalarning birlashmasi B ni o'z ichiga

oladigan ochiq B' to‘plamni tashkil etadi. Agar 2 ta $L(a_i,\rho)$ va $L(a_j,\rho)$ lemniskatalar kesishsa, u holda $|\psi(a_i,a_j)|<\frac{\rho_i+\rho_j}{2}$ dan kelib chiqadiki, $|\psi(a_i,a_j)|<\max\{\rho_i,\rho_j\}$ bo‘ladi. Aytaylik, $\max\{\rho_i,\rho_j\}=\rho_i$ bo‘lsin. U vaqtda $L(a_i,\rho)$ lemniskata a_j nuqtani o‘z ichiga oladi, binobarin, B to‘plamning nuqtalari cheksiz to‘plam bo‘ladi. $\psi(z,a_i)$ va $\psi(z,a_j)$ bo‘yicha yoyilgan qatorlar ko‘rsatilgan to‘plamning nuqtalarida ustma-ust tushadi, shuningdek, $L(a_i,\rho)$ va $L(a_j,\rho)$ larning kesishadigan umumiy qismida bu qatorlar bir xil bo‘ladi. Shunday qilib, $\sum_{k=0}^{\infty}c_j^{(k)}\psi^k(z,a_j),\ j=1,2,...,p$ darajali qatorlar $f(z)$ funksiya mos lemniskatalarda ifodalanadi, B' da aniqlangan $A(z)-$analitik funksiyalar B da $f(z)$ bilan ustma-ust tushadi. Agar B' juft-jufti bilan umumiy nuqtasi bo‘lmaydiga qavariq sohalarning birlashmasidan iborat bo‘lsa, u holda B' to‘plamga va $f(z)$ funksiyalarga yuqoridagi $A(z)-$analitik funksiyalar uchun Runge teoremasini qo‘llash qoldi. B' sohaning komponentali to‘plamlari orasida ko‘p bog‘lamli sohalari bo‘lsin. B' to‘plamning chegarasi chekli p sondagi kontinum quvvatli to‘plamdan tuzilgan, E juft-jufti bilan umumiy nuqtaga ega bo‘lmagan va E sohada joylashgan. E sohaning ichida $\zeta_1,\ \zeta_2,\ ...,\ \zeta_p$ larning har bir nuqta tanlanishida $\zeta_1,\ \zeta_2,\ ...,\ \zeta_{p-1}$ bilan ζ_p nuqtalarning jordan $\gamma_1,\ \gamma_2,\ ...,\ \gamma_p$ yoylari bilan tutashtiramiz. U vaqtda $B'\setminus(\gamma_1\setminus\gamma_2\setminus...\setminus\gamma_{p-1})=D$ ochiq to‘plamni ifodalaydi, B ni va umumiy nuqtalarga ega bo‘lmagan barcha komponentali qavariq sohalarni o‘z ichiga oladi. $B\subset D\subset D'$ ni qo‘shishni qoldiramiz, shunday qilib Runge teoremasi analogini qo‘llash shartlarida ko‘rsatamiz.

Foydalanilgan adabiyotlar ro‘yxati:

(1). Маркушевич А.И. Теория аналитических функций, Москва, Государственное издательство технико-теоретической литературы, 1950, 704 с.

(2). Векуа И.Н. Обобщённые аналитические функции, Москва, “Наука”, 1988, 512 с.

(3). Жабборов Н.М., Отабоев Т.У. Теорема Коши для $A(z)-$аналитических функций, Узб. мат. жур., 2014, № 1, с. 15–18.

(4). Жабборов Н.М., Отабоев Т.У. Аналог интегральной формулы Коши для $A(z)-$аналитических функций, Узб. мат. жур., 2016, № 4, с. 50–59.

(5). Sadullayev A., Jabborov N.M. On a class of $A-$analytic functions. Journal Siberian Federal University, Mathematics & Physics, Volume 9. Issue 3. 2016, 374–383 p.

Ismaylov Qanatbay Abdureyimovich (Qoraqalpoq davlat universiteti professori),
Yakubov Komiljon Ruzmetovich (Abu Rayhon Beruniy nomidagi Urganch davlat universiteti dotsenti, professor v.b.),
Qurbanov Davron Shavkatovich (Abu Rayhon Beruniy nomidagi Urganch davlat universiteti, katta o‘qituvchi; davronkurbanow89@gmail.com),
Qadambayeva Raviyajon Bakpulatovna (Urganch davlat universiteti magistarnti),
Aminov Doniyorbek Rashidovich (Urganch davlat universiteti doktoranti)

KREMNIY ASOSLI p-Si/n-Si, p^+-Si BSF/p-Si/n-Si VA p^+-Si BSF/p-Si/ZnO QUYOSH ELEMENTLARINING ELEKTROFIZIK XUSUSIYATLARINI O‘RGANISH

Annotatsiya. *Mazkur maqolada kremniy asosidagi p-Si/n-Si, p^+-Si BSF/p-Si/n-Si hamda p^+-Si BSF/p-Si/ZnO quyosh elementlarining elektrofizik xususiyatlari nazariy jihatdan tadqiq qilindi. Tadqiqot davomida turli qatlam tuzilmalari uchun asosiy fotoelektrik parametrlarning o‘zgarishi tahlil etildi. Natijalar shu-*