Conclusion. In this paper, we have studied the second-order asymptotic behaviour of the survival probability of a critical Bellman-Harris process above the heavy-tail threshold. Under Slack's regular-variation condition for the reproduction law and regular variation of the lifetime tail, we have described the structure of the remainder beyond the classical leading asymptotic term.The analysis shows that the second-order behaviour is determined by two distinct mechanisms. The first mechanism is renewal-driven and is associated with the equilibrium tail of the lifetime distribution. The second is local and is represented by the Stone constant of the renewal density. The comparison of these two mechanisms gives rise to a second threshold, which determines the dominant form of the correction term.Three principal regimes are obtained. In one regime, the renewal-driven contribution produces a power-type correction. In another, the corresponding integral is absorbed into the constant and the logarithmic term becomes the leading second-order contribution. At the balance point, the two mechanisms occur on the same scale and their contributions combine, producing a logarithmic correction with the coefficients of both mechanisms. Thus, the logarithmic term remains an essential component of the second-order structure throughout the considered range.A central feature of the approach is that the entire second-order analysis is reduced to the two-term asymptotics of a single convolution obtained from an exact identity for the survival probability. This provides a unified framework for describing the different asymptotic regimes and clarifies the roles of the renewal-driven and local mechanisms.

Finally, the results are consistent with the known finite-variance case. In the appropriate special regime, the obtained expansion reduces to the previously established remainder expansion for the nonlinear renewal equation. Therefore, the results extend the known finite-variance theory to the more general setting of regularly varying reproduction laws and heavy-tailed lifetime distributions.

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DEPENDENCE OF THE ELECTRICAL CONDUCTIVITY AND TENZORESISTIVE PROPERTIES OF POLYCRYSTALLINE PbSe FILMS ON THE TECHNOLOGICAL PARAMETERS

Annotation. *The influence of technological parameters on the electrical conductivity and strain-sensitive properties of polycrystalline PbSe films was investigated. The effects of substrate temperature, deposition position, and film thickness on charge transport were analyzed. Changes in these parameters were found to affect electrical conductivity and may cause a transition from p-type to n-type conductivity. This behavior is attributed to compositional inhomogeneity, grain-boundary barriers, and charge-carrier redistribution. Optimal technological conditions provide enhanced electrical conductivity and strain sensitivity of PbSe films.*

Keywords: *PbSe, polycrystalline films, electrical conductivity, strain sensitivity, piezoresistive effect, substrate temperature, film thickness, conductivity-type inversion, technological factors.*

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POLIKRISTAL PbSe PLYONKALAR ELEKTR O'TKAZUVCHILIGI VA TENZOMETRIK XUSUSIYATLARINING TEXNOLOGIK PARAMETRLARGA BOG'LIQLIGI

Annotatsiya. *PbSe polikristall plyonkalarining elektr o'tkazuvchanligi va tenzosezgirlik xossalariga texnologik parametrlarning ta'siri o'rganildi. Taglik harorati, cho'ktirish holati va plyonka qalinligining zaryad tashuvchilar ko'chishiga ta'siri tahlil qilindi. Ushbu parametrlarning o'zgarishi elektr o'tkazuvchanlikka ta'sir qilishi hamda p-turdan n-tur o'tkazuvchanlikka o'tishni yuzaga keltirishi aniqlandi. Kuzatilgan holatlar tarkibiy bir jinslilikning buzilishi, donalar chegarasidagi potensial to'siqlar va zaryad tashuvchilarning qayta taqsimlanishi bilan izohlanadi. Optimal texnologik sharoitlar PbSe plyonkalarining elektr o'tkazuvchanligi va tenzosezgirligini oshirish imkonini beradi.*

Kalit so'zlar: *PbSe, polikristall plyonka, elektr o'tkazuvchanlik, tenzosezgirlik, pyezorezistiv effekt, taglik harorati, plyonka qalinligi, o'tkazuvchanlik turi inversiyasi, texnologik omillar.*

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ЭЛЕКТРОПРОВОДНОСТЬ И ТЕНЗОСВОЙСТВА ПОЛИКРИСТАЛЛИЧЕСКИХ ПЛЕНОК PbSe В ЗАВИСИМОСТИ ОТ ТЕХНОЛОГИЧЕСКИХ ПАРАМЕТРОВ

Аннотация. *Исследовано влияние технологических параметров на электропроводность и тензочувствительные свойства поликристаллических пленок PbSe. Проанализировано влияние температуры подложки, положения осаждения и толщины пленки на перенос носителей заряда. Установлено, что изменение этих параметров влияет на электропроводность и может вызывать переход от p-типа к n-типу проводимости. Наблюдаемое поведение связано с неоднородностью состава, потенциальными барьерами на границах зерен и перераспределением носителей заряда. Оптимальные технологические условия обеспечивают повышение электропроводности и тензочувствительности пленок PbSe.*

Ключевые слова: *PbSe, поликристаллические пленки, электропроводность, тензочувствительность, пьезорезистивный эффект, температура подложки, толщина пленки, инверсия типа проводимости, технологические факторы.*

An important factor affecting the strain-sensitive properties of thin-film elements is the degree of deviation from the stoichiometric composition during evaporation. Thermodynamic data indicate that Se has a higher volatility than Pb, and recent investigations of polycrystalline PbSe films have confirmed that elevated processing temperatures can enhance Se loss and increase the deviation of the Pb/Se atomic ratio from stoichiometry [7]. As shown by our investigations, the enrichment of the solid phase with lead is associated with the recrystallization of PbSe powder.

Lead selenide films deposited on amorphous substrates within the temperature range of 50–150 °C exhibit a polycrystalline structure consisting of randomly oriented grains. With an increase in the substrate temperature up to 100 °C, the electrical conductivity of the films increases. Within this temperature range, the films exhibit p-type conductivity. At $t_n > 100^0 C$, however, the specific electrical conductivity σ decreases, and the conductivity type of the films changes to n-type (Fig. 1).

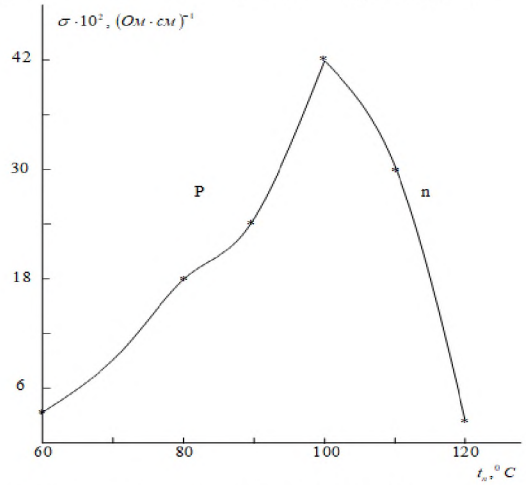


Fig. 1. Dependence of the electrical conductivity of PbSe films on the substrate temperature.

Taking into account that, in the investigated films, vacancies in the metal sublattice act as acceptors, whereas chalcogen vacancies act as donors, it can be assumed that an increase in the condensate deposition temperature leads to the "healing" of metal vacancies in the cation sublattice of the material. This may occur due to the enrichment of the condensate with selenium at high deposition temperatures, owing to the significant difference between their saturated vapor pressures [1].

Changes in the electrical conductivity of PbSe films deposited at different substrate temperatures are accompanied by changes in the charge-carrier concentration. Recent Hall-effect studies have also demonstrated that thermal processing and defect passivation can substantially modify both carrier concentration and carrier mobility in PbSe films [10]. Lead selenide films obtained within the above-mentioned substrate-temperature range exhibited carrier concentrations on the order of 10^{16}-10^{17} sm^{-3}. The electrical conductivity of the films measured from the current–voltage (I–V) characteristics, as well as the complex conductivity measured using the microwave technique, showed a correlated behavior. The qualitative agreement between the electrical conductivity measured under direct current conditions and that measured at a frequency of $5.1 \cdot 10^{10}$ Hz, as a function of substrate temperature, indicates that an increase in substrate temperature changes the charge-carrier concentration and is accompanied by an inversion of the conductivity type (Fig. 1).The experimental results show that the electrical conductivity of PbSe films depends on the substrate temperature, the position of the deposition region, the film thickness, and the deposition rate.

Figure 2 shows the dependence of the electrical conductivity, σ, of PbSe films on the substrate position. The curve represents the variation in σ measured under direct-current conditions.

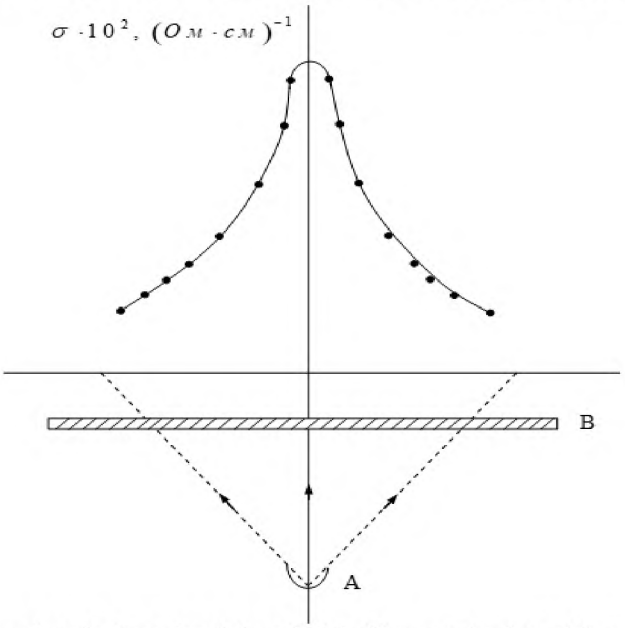


Fig. 2. Dependence of the electrical conductivity of PbSe films on the deposition position: A – evaporation boat; B – substrate.

The main parameter in such an experiment is the composition gradient or concentration gradient along a certain direction on the substrate. This assumption is supported by the results of spectral analysis of the film composition at different positions on the substrate.

Charge transport in deposited semiconductor films is governed by the inhomogeneous properties of the condensate and by charge-carrier scattering at grain boundaries. The influence of the thickness of inhomogeneous films on the kinetic properties of lead chalcogenides is explained by the formation of potential barriers at grain boundaries [2,3]. For PbSe films, an increase in the substrate temperature promotes the formation of continuous films and, consequently, increases the substrate coverage factor β, defined as the ratio of the area occupied by the condensate to the total substrate area. An increase in the substrate temperature up to 90 °C (Fig. 1), at a constant deposition rate, leads to an increase in the electrical conductivity σ. This indicates that increasing the substrate temperature results in more intensive growth of the condensate at randomly distributed crystallization centers [4].

Figure 3 shows the dependence of the electrical conductivity, σ, of PbSe films on film thickness. As can be seen from the figure, the thickness of polycrystalline PbSe films is an important parameter affecting not only the magnitude of σ, but also the conductivity type.

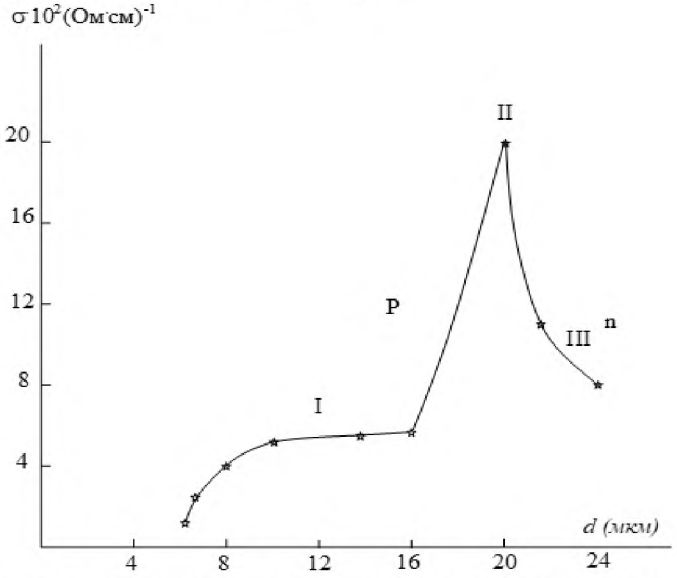


Fig. 3. Dependence of the electrical conductivity of PbSe films on film thickness.

Recent investigations have similarly demonstrated that variations in PbSe film thickness and stoichiometric composition can be accompanied by substantial changes in surface resistivity, indicating a close relationship between film growth, composition, and charge transport [8].

This phenomenon is associated with the interaction of lead ions moving toward the film surface with oxygen. As a result, the surface layers acquire p-type conductivity, whereas the deeper layers exhibit n-type conductivity [6]. Since the rate of interaction between oxygen and lead cannot be uniform over the entire film surface, the region with p-type conductivity does not form a continuous surface layer but instead has an island-like character. The presence of such inhomogeneities in the film inevitably leads to the formation of p–n junctions within the film, oriented both parallel and perpendicular to its surface.

The dependence of electrical conductivity on film thickness was investigated in an air atmosphere, and three characteristic regions were observed. In the first region, the samples exhibit p-type conductivity, and the electrical conductivity depends only weakly on film thickness.

In the second region, corresponding to a thickness of 17–20 μm, an inversion of the conductivity type is observed. At $d<17\ \mu m$, the films exhibit p-type conductivity, whereas at $d>20\ \mu m$, they exhibit n-type conductivity. In the same thickness range, a sharp increase in electrical conductivity is observed. In the third region, the electrical conductivity decreases.

It was established that, in PbSe films with a thickness of approximately 1 μm, the observed charge-carrier mobility is considerably lower than that in bulk single crystals and is proportional to the film thickness. This difference is consistent with the strong influence of grain boundaries on carrier transport in PbSe; recent studies on high-quality PbSe crystals have demonstrated that suppressing grain-boundary scattering can lead to markedly enhanced carrier mobility [9].

To investigate the spatial character of charge-carrier transport in the film, measurements of the Hall electromotive force (Hall EMF) are required. Hall-effect measurements in PbSe have been reported in a large number of studies, a list of which is given in monograph [5]. The obtained results indicate that the high strain-sensitivity coefficient is caused by the redistribution of charge carriers in inhomogeneous films. The analysis also shows that there is an optimum film thickness at which the strain-sensitivity coefficient passes through an extremum.

Conclusion. Thus, the results obtained from measurements of the electrical conductivity under direct-current conditions and the complex electrical conductivity measured under alternating-current conditions showed satisfactory agreement in their dependence on substrate temperature.

The conductivity type of PbSe films can be controlled by changing the substrate temperature without changing the initial source material. Under optimum technological conditions, the electrical conductivity and strain sensitivity reach their maximum values.

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SPECTRAL ANALYSIS OF THE FIBER SCHRÖDINGER OPERATORFOR A TWO-BOSON SYSTEM ON THE TWO-DIMENSIONAL LATTICE

Annotation. *We study the fiber Schrödinger operator for a two-boson system on the two-dimensional lattice with on-site, nearest-neighbor, and axial second nearest-neighbor interactions. At zero total quasi-momentum the operator is reduced to the coordinate-exchange-symmetric subspace, where the interaction has rank three. Using the Lippmann–Schwinger operator, the eigenvalue problem is reduced to a finite-dimensional Fredholm determinant. We derive the threshold asymptotics of this determinant, describe the corresponding critical sets in the interaction-parameter space, and prove that these sets divide the parameter space into regions on which the number of discrete eigenvalues is constant. This yields a complete classification of the number of eigenvalues below and above the essential spectrum.*

Keywords: *two-boson system, lattice Schrödinger operator, fiber operator, essential spectrum, Fredholm determinant, discrete spectrum.*

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IKKI O‘LCHAMLI PANJARADAGI IKKI BOZONLI SISTEMAGA MOS QAVAT SHRYODINGER OPERATORINING SPEKTRAL TAHLILI

Annotatsiya. *Biz ikki o‘lchamli panjaradagi, bir tugundagi, eng yaqin qo‘shni va o‘q yo‘nalishidagi ikkinchi qo‘shni o‘zaro ta’sirlarga ega ikki bozonli sistema uchun qavat Shrödinger operatorini o‘rganamiz. To‘la kvaziimpuls nol bo‘lganda operator koordinatalarni almashtirishga nisbatan simmetrik qism fazoga keltiriladi, bunda o‘zaro ta’sir operatorining rangi uchga teng. Lippmann–Shwinger operatori yordamida xos qiymatlar masalasi chekli o‘lchamli Fredholm determinantiga keltiriladi. Ushbu determinantning spektr chegaralaridagi asimptotikalarini topamiz, o‘zaro ta’sir parametrlari fazosidagi mos kritik to‘plamlarni tavsiflaymiz va bu to‘plamlar parametrlar fazosini diskret xos qiymatlar soni o‘zgarmas bo‘lgan sohalarga ajratishini isbotlaymiz. Natijada muhim spektrdan pastda va yuqorida joylashgan xos qiymatlar sonining to‘liq tasnifi olinadi.*